Multiple Nested Structures: the Curse (or Blessing?) of Applied Mathematics

Antonio Frangioni[1]

[1]Dipartimento di Informatica, Università di Pisa

Calcolo Scientifico e Modelli Matematici:
Alla Ricerca delle Cose Nascoste
Attraverso le Cose Manifeste 2.0
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Outline

1. Structure: top-down
2. Structure: bottom-up
3. The Core Elements of `SMS++`
4. The quasi-Core Elements of `SMS++`
5. Example: `SMS++` for the Unit Commitment
6. Conclusions and (a Lot of) Future Work

Structure in Optimization Problems

- General optimization problem $(P)\ \min\{\, c(x) \,:\, x \in X \,\}$: clearly unsolvable if $c(\cdot)$ and X are "any" function/set
- To do anything one needs assumptions on the structure of $c(\cdot)/X$
- Many different cases, most of them hard
- Let's take it easy: strong structure $\equiv$ easy problem:

  Linear Program $(P)\ \min\{\, cx \,:\, Ax = b \ ,\ 0 \le x \le u \,\}$

  $A \in \mathbb{R}^{n \times m}$ (sparse), $b \in \mathbb{R}^n$, $c \in \mathbb{R}^m$, $u \in \mathbb{R}^m$, $m > n$
- Structure $\Longrightarrow$ useful properties: dual problem to (P)

  $(D)\ \max\{\, yb - wu \,:\, yA + z - w = c \ ,\ z \ge 0 \ ,\ w \ge 0 \,\}$

  fundamental tool for solving (P)

Solving LPs is (Structured, non)Linear Algebra

- Karush-Kuhn-Tucker optimality conditions ($diag(V) = v$, $e =$ all 1s)

$$(KKT) \left\{ \begin{array}{ll} Ax = b \;,\; x + s = u \;,\; yA + z - w = c & \text{(linear)} \\ XZe = 0 \;,\; SWe = 0 & \text{(nonlinear)} \\ [\, x \,,\, s \,,\, z \,,\, w \,] \geq 0 & \text{(inequalities)} \end{array} \right.$$

- Interior Point methods for LP: "slacken + linearize":

  i) $\mu > 0$, $(KKT_\mu) \equiv (KKT)$ except $XZe = \mu e \;,\; SWe = \mu e$
  $\Longrightarrow$ $(2m\mu)$-optimal solution

  ii) feasible $[\, \bar{x} \,,\, \bar{s} \,,\, \bar{z} \,,\, \bar{w} \,] > 0$, $v = \bar{v} + \Delta v$ (stepsize ensures $v > 0$) $\Longrightarrow$

$$\left\{ \begin{array}{l} A\Delta x = 0 \;,\; \Delta x + \Delta s = 0 \;,\; \Delta yA + \Delta z - \Delta w = 0 \\ \bar{X}\bar{Z}e + \bar{X}\Delta z + \bar{Z}\Delta x = \mu e \;,\; \bar{S}\bar{W}e + \bar{S}\Delta w + \bar{W}\Delta s = \mu e \end{array} \right.$$

  ignore second-order terms $\equiv$ Newton's method for nonlinear equations

- $[\, \bar{x} \,,\, \bar{s} \,,\, \bar{z} \,,\, \bar{w} \,]$ satisfies (KKT_μ): one iteration, $\mu \searrow$ (fast), repeat
- Many improvements (infeasible method, predictor corrector, ...)

Solving LPs is Structured Linear Algebra

- Boils down to Reduced KKT or Normal equations ($\Theta > 0$ diagonal)

$$\begin{bmatrix} -\Theta & A^T \\ A & 0 \end{bmatrix} \qquad\qquad A\Theta A^T$$

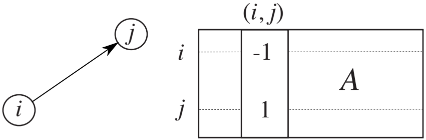
$m + n \times m + n$, sparse $\qquad$ $n \times n$, a lot less sparse

- Special case of Saddle-Point system, lots of applications (physics, engineering, economy, computer science, ...), very active research[1]
- Specific twists in the LP case:
  - large size: $m \approx 10^{6+}$, $n \approx 10^{5+}$ ...
  - must be solved many times, but rather inexactly (at the first iterations)
  - fixed nonzero structure (A) and variable data (Θ)
  - special evolution of data over the iterations
  - no discretization, no underlying smooth operator
- Ultimate performances require assumptions on (structure of) A

[1] Benzi, Golub, Liesen "Numerical solution of saddle point problems" *Acta Numerica* 14, 1–137, 2005

Going Deeper: More Assumptions ($\equiv$ Structure)

- A = *node-arc incidence matrix* of directed graph G



$\equiv$ min-cost flow problem
$\equiv$ $A\Theta A^T$ weighted Laplacian of G (an M-matrix)

- Can tell a lot on the system by looking at the graph[2,3]
- Can do a lot about the system by working on the graph:
  - preconditioners are (chordal) sub-graphs, can be obtained by efficient graph algorithms (Kruskal[4], Prim[5], ...)
  - projection in algebraic multigrid is merging nodes[6]
  - projection and preconditioning is a unique graph-based process[7]

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2 Cvetković, Doob, Sachs "Spectra of graphs", 1980 — Brouwer, Haemers "Spectra of Graphs", 2012

3 F., Serra Capizzano "Spectral Analysis of (Sequences of) Graph Matrices" *SIMAX*, 2001

4 F., Gentile "New Preconditioners for KKT Systems of Network Flow Problems" SIOPT, 2004

5 F., Gentile "Prim-based Support-Graph preconditioners for Min-Cost Flow Problems" *CO&A*, 2006

6 Dell'Acqua, F., Serra Capizzano "Computational Evaluation of Multi-Iterative Approaches [...]" *CALCOLO*, 2015

7 Dell'Acqua, F., Serra Capizzano "Accelerated Multigrid for Graph Laplacian Operators" *Appl. Math. & Comp.*, 2015

Lots of Fun for Lots of Different People

- These systems have been approached by many different angles:
  - graph theory
  - computer science
  - numerical linear algebra
  - optimization
  - physics . . .

- Lots of ingenuity, theoretical results, implementations

- Applied mathematics at its best: focus on one structure with relevant applications, drill it down until it cries

Lots of Fun for Lots of Different People

- These systems have been approached by many different angles:
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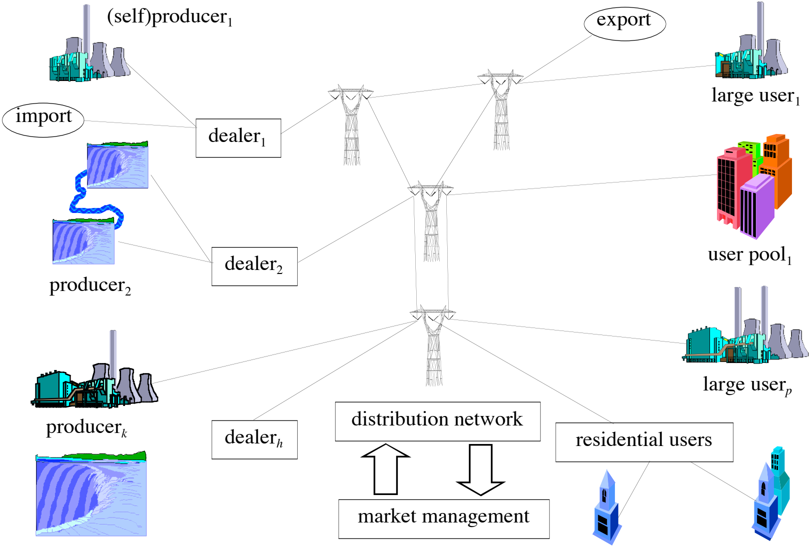
- Lots of ingenuity, theoretical results, implementations
- Applied mathematics at its best: focus on one structure with relevant applications, drill it down until it cries
- Is this always enough?

Outline

Motivation: The Unit Commitment (UC) problem

- Schedule a set of generating units $\mathcal{U}$ over a set of time instants $\mathcal{T}$ to satisfy the (forecasted) demand d_t at each $t \in \mathcal{T}$



The Unit Commitment problem

- Gazzillions €€€ / $$$, enormous amount of research[8,9]

- What has it to do with networks? More than it would seem

- Different types of production units, different constraints:
  - Thermal (comprised nuclear): min/max production, min up/down time, ramp rates on production increase/decrease, start-up cost depending on previous downtime, others (modulation, ...)
  - Hydro (valleys): min/max production, min/max reservoir volume, time delay to get to the downstream reservoir, others (pumping, ...)
  - Non programmable (ROR hydro) intermittent units (solar/wind, ...)
  - Fancy things (small-scale storage, demand response, smart grids, ...)

- Plus the interconnection network (AC/DC, transmission/distribution)

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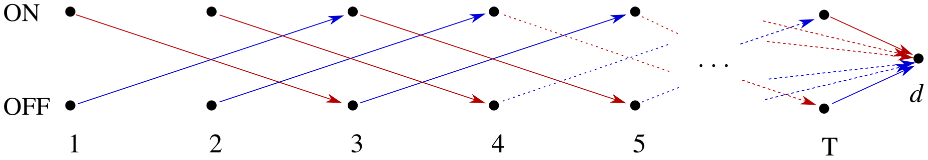
8 van Ackooij, Danti Lopez, F., Lacalandra, Tahanan "Large-scale Unit Commitment Under Uncertainty [...]" *AOR*, 2018

9 The plan4res project: https://www.plan4res.eu/

- Again, what did this have to do with graphs, please?
- Specialized DP algorithms for thermal single-Unit Commitment[10]

  $\equiv$ shortest path on appropriate acyclic graph



- Not too many nodes $2(T = |\mathcal{T}|)$, but rather dense: $O(T^2)$ arcs
- $(\,(\,t\,,\mathrm{ON}\,)\,,(\,\tau\,,\mathrm{OFF}\,)\,) \equiv$ startup at t and shutdown at $\tau > t$ . . .
- Costs require another nested DP per arc, $O(T^3)$ overall
- Hence, (strong but large) formulation as a flow problem[11]

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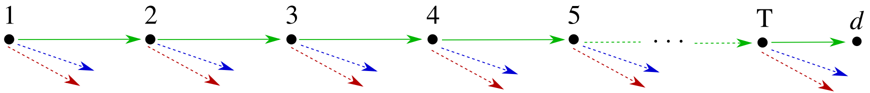
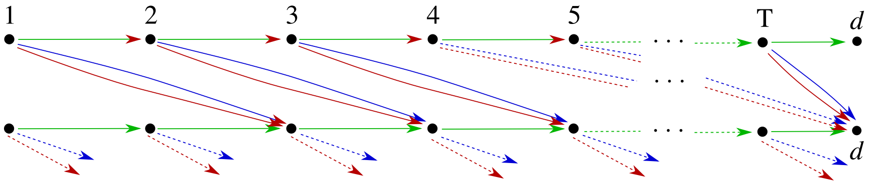
[10] F., Gentile "Solving Nonlinear Single-Unit Commitment Problems with Ramping Constraints" *OR*, 2006

[11] F., Gentile "New MIP Formulations for the Single-Unit Commitment Problems with Ramping Constraints" *IASI 15-06*, 2015

- Water flowing over time is a flow problem (surprise!)



- Quite skinny graph, $O(T)$ nodes and arcs, too
- Turbining/spilling arcs produce/not energy (max reservoir capacity)
- However, hydro units are often whole interconnected hydro valleys



- All in all (without pumping) still a flow problem, on a structured graph (composition of lines with a reverse tree)

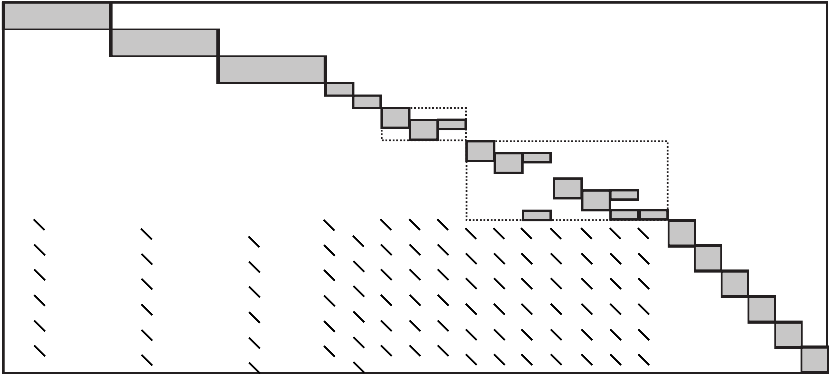
The Network

- The transmission/distribution network is a graph (surprise!)
  nodes are zones/buses, arcs are links (bi-directed)
- Kirchhoff's current law: $Af = n$ ($f =$ flows, $n =$ net injection)
- Kirchhoff's voltage law + Ohm's law for AC current $\Longrightarrow f = \gamma^T A^T \theta$
  ($\theta =$ voltage angles, $\gamma =$ arc susceptances $= 1/$ impedence)
- AC $\Longrightarrow$ currents and voltages are periodic $\equiv$ complex numbers
- DC approximation: $|\, \theta_i - \theta_j \,| \ll 1$ $(i, j) \in A$ (small phase differences between neighbours) $\Longrightarrow$ can linearize the trigonometric functions
- $A \Gamma A^T \theta = n$ (Laplacian!) + $\underline{f} \leq \gamma^T A^T \theta \leq \bar{f}$ (capacity)
- Fixing one reference voltage $A \Gamma A^T$ nonsingular:

  $$\underline{f} \leq \gamma^T A^T (A \Gamma A^T)^{-1} n \leq \bar{f}$$
- True AC version nonlinear nonconvex, rater hard ...

Putting it all together

- Not a single flow but a multicommodity flow (of sorts)



- Many blocks, either A or $A\Gamma A^T$, but of rather different shape and size
- Nontrivial linking constraints

Can We Deal With Such a Structure?

- Of course we can, in fact with several different approaches:
  - Lagrangian decomposition[12] and related methods[13], even in parallel[14]
  - Structured Interior-Point methods[15]
  - Structured active-set (simplex) methods[16]
  - Structured Dantzig-Wolfe decomposition[17,18]
  - ...
- Most can exploit the "inner" graph structure of (the many) A(s)
- Significantly more complex: two-level approaches ($\equiv$ more fun)

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12 F., Gallo "A Bundle type dual-ascent approach to linear multicommodity min cost flow problems" *INFORMS JOC*, 1999

13 Grigoriadis, Khachiyan "An exponential function reduction method for block angular convex programs" *Networks*, 1995

14 Cappanera, F. "Symmetric and asymmetric parallelization of a cost-decomposition algorithm [...]" *INFORMS JOC*, 2003

15 Castro "Solving difficult multicommodity problems through a specialized interior-point algorithm" *Ann. OR*, 2003

16 McBride "Progress made in solving the multicommodity flow problem" *SIOPT*, 1998

17 F., Gendron "A stabilized structured dantzig-wolfe decomposition method" *Math. Prog.*, 2013

18 Mamer, McBride "A decomposition-based pricing procedure for large-scale linear programs [...]" *Man. Sci.*, 2000

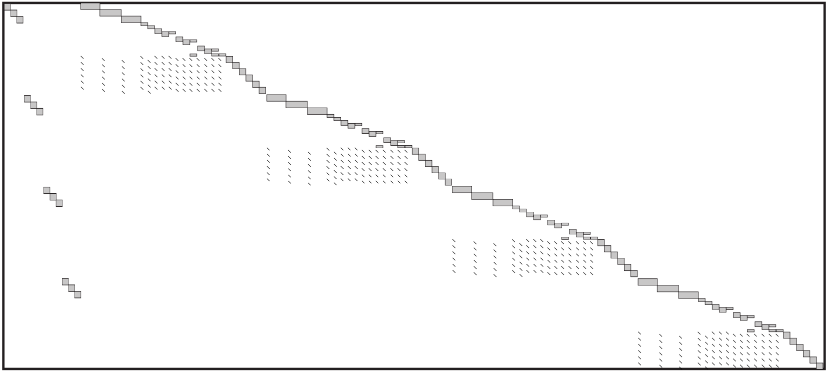
Is All Well in Structure Land, Then?

- Maybe if this were the end, but it is just the beginning
- Data is uncertain: demand, wind/solar production, units/network state ... which cannot be ignored (increased RES penetration ...)
- Unit commitment is decided in advance (here-and-now) but actual dispatch can be changed in real time (recourse)
- Many methods to represent uncertainty: Stochastic Optimization[19], Robust Optimization, Chance-Constrained Optimization, hybrid[20]
- Simplest approach scenario-based: each $\approx$ a full UC

  $\Longrightarrow$ yet another two-level structure
- Cons: size increases of a factor # scenarios (which should be large)

[19] Scuzziato, Finardi, F. "Comparing Spatial and Scenario Decomposition for Stochastic [...]" *IEEE Trans. Sust. En.*, 2018
[20] van Ackooij, F., de Oliveira "Inexact Stabilized Benders' Decomposition Approaches, with Application [...]" *CO&A*, 2016

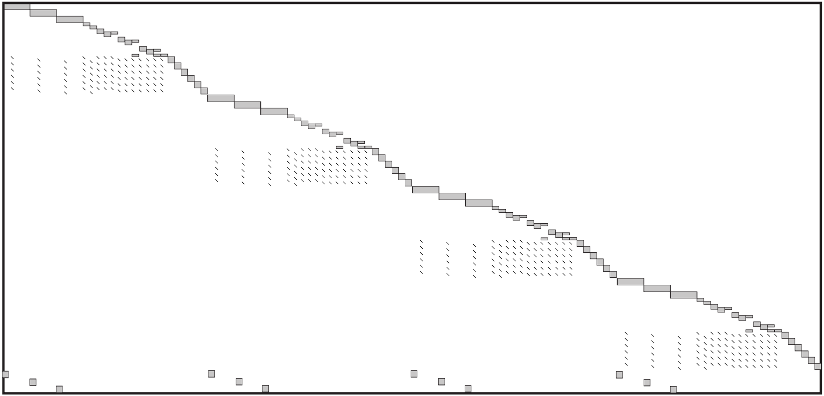
Scenario-based Structure



- Perfect structure for Benders' decomposition
- Benders' decomposition with Lagrangian decomposition inside[21]
- ... with (different) graph structure(s) inside

[21] van Ackooij, Malick "Decomposition algorithm for large-scale two-stage unit-commitment" *Ann. OR*, 2016

An Aside (not really): Reformulation



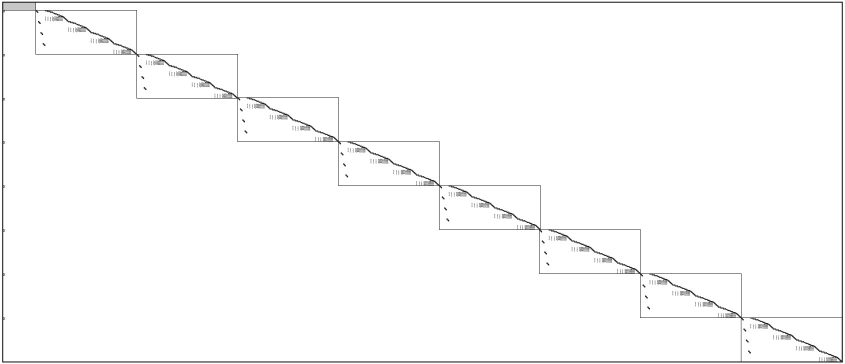
- Or was it the perfect structure for Lagrangian decomposition?
- Lagrangian decomposition with Lagrangian decomposition inside ...
- Which is better? Very hard to say beforehand[19]

OK, This is Two-Level Decomposition, Then?

- Unit-Commitment is a short-term problem, lacks long-term strategies
- Issue: cost of water (none) / minimum reservoir volume (very low) $\Longrightarrow$ lot of water used $\Longrightarrow$ no water most of the year
- Hydro production most useful for peak shaving every day
- Computing value of water left in the reservoirs at T
  $\equiv$ solving a parametric (uncertain) UC problem
  for each (significant) day of the year
- Can approximate it by dual variables/Lagrangian multipliers of minimum reservoir volume constraints
- Better a piecewise linear representation (cutting-plane model)
- Then, stochastic dual dynamic programming[22] (another graph)

[22] Pereira, Pinto "Multi-stage stochastic optimization applied to energy planning" *Math. Prog.*, 1991

Complete Tactical Problem

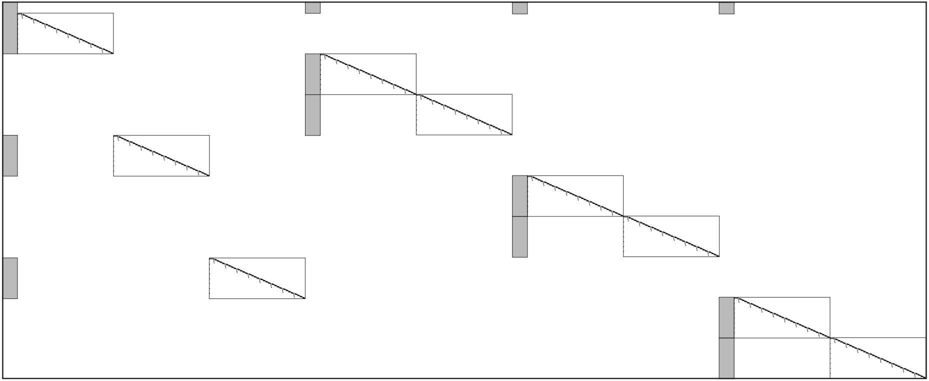


- This is not really how you'd do that (integer variables)
- Still OK for Benders-like decomposition
- Benders + Benders + Lagrange + Graph or
  Benders + Lagrange + Lagrange + Graph or
  Lagrange + Benders + Lagrange + Graph or ...

OK, But This Surely is The End, Right?

- The energy system changes all the time, but modifications slow, extremely costly, with huge inertia
- Demand and production subject to very significant uncertainties: climate = RES production + demand, shifts in consumption patterns (EV, cryptocurrencies, . . .), new technologies (shale, LED, . . .), geo-political factors (energy security), economical factors (boost or boom), regulatory factors (EU energy market, . . .), political factors (CO_2 emission treaties, nuclear power, . . .), . . .
- Planning long-term evolution very hard, yet necessary
- 20/30 years, 2/5 years steps (multi-level recourse), many scenarios

Complete Strategic Problem



- Huge size, multiple nested structure
- Still OK for either Benders or Lagrange
- Benders + Lagrange + Benders + Lagrange + Graph or . . .

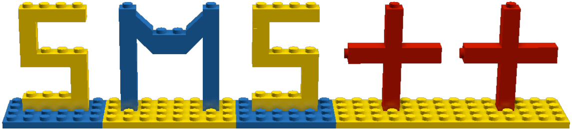
How Do you Solve Such a Thing?

- Modeling system: easily construct a huge, flat = unstructured matrix to be passed to a general-purpose, flat solver
- Some solvers offer one-level decomposition (Benders, CG = DW)
- Attempts at automatically recovering structure from a matrix[23], but only one level and anyway conceptually awkward
- Only one tool (that I know of) for multiple nested structure[24,25], but only solves continuous problems by Interior Point methods
- Nothing for multilevel, heterogeneous approaches (such as, but not only, decomposition), e.g., allowing specialized solvers for each block
- So far

[23] Gamrath, Lübbecke "Experiments with a Generic Dantzig-Wolfe Decomposition for Integer Programs" *LNCS*, 2010
[24] Gondzio, Grothey "Exploiting Structure in Parallel Implementation of Interior Point Methods [...]" *Comput. Man. Sci.*, 2009
[25] Colombo et al. "A Structure-Conveying Modelling Language for Mathematical [...] Programming" *Mathe. Prog. Comp.*, 2009

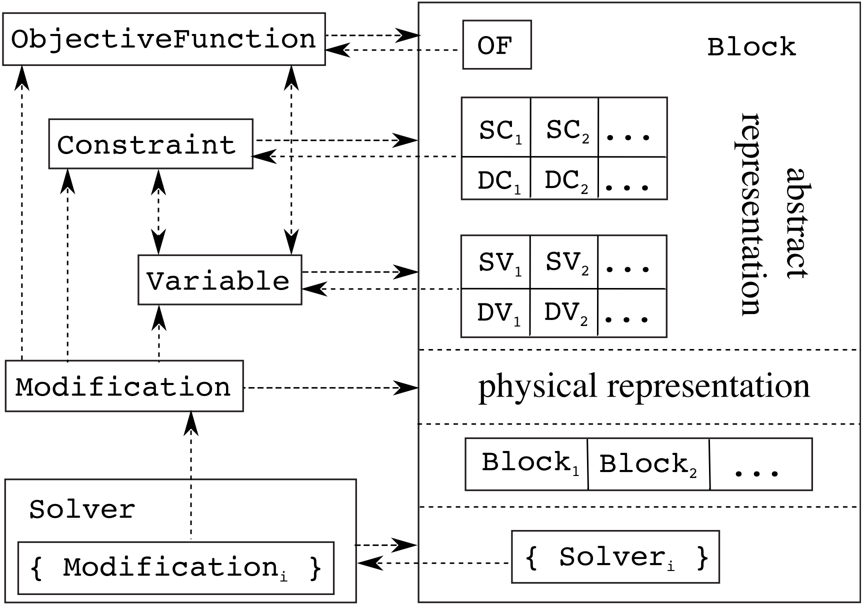
Outline

What We Want



- A modelling language/system which:
  - explicitly supports the notion of block ≡ nested structure
  - separately provides "semantic" information from "syntactic" details (list of constraints/variables)
  - allows exploiting specialised solvers on blocks with specific structure
  - caters all needs of complex methods: dynamic generation of constraints/variables, modifications in the data, reoptimization
- C++ library: set of "core" classes, easily extendable
- Why C++? A number of reasons:
  - all serious solvers are written in C/C++
  - we all love it (especially `C++11/14`)
  - tried with Julia/JuMP, but could not handle well C++ interface

The Core SMS++



ObjectiveFunction
Constraint
Variable
Modification
Solver
{ Modification$_i$ }
OF
Block
SC_1 SC_2 ...
DC_1 DC_2 ...
SV_1 SV_2 ...
DV_1 DV_2 ...
abstract representation
physical representation
$Block_1$ $Block_2$ ...
{ $Solver_i$ }

Block

- Block = abstract class representing the general concept of "a part of a mathematical model with a well-understood identity"
- Each Block:: a model with specific structure (e.g., Block::BinKnapsackBlock = a 0/1 knapsack problem)
- Physical representation of a Block: whatever data structure is required to describe the instance (e.g., *a*, *b*, *c*)
- Abstract representation of a Block:
  - one (for now) ObjectiveFunction
  - any # of groups of (pointers) to (static) Variable
  - any # of groups of std::list of (pointers) to (dynamic) Variable
  - any # of groups of (pointers) to (static) Constraint
  - any # of groups of std::list of (pointers) to (dynamic) Constraint

  groups of Variable/Constraint can be single (std::list) or std::vector (...) or boost::multi_array thanks to boost::any
- Any # of sub-Blocks (recursively), possibly of specific type (e.g., Block::MMCFBlock can have *k* Block::MCFBlocks inside)

- Abstract concept, thought to be extended (a matrix, a function, ...)
- Does not even have a value
- Knows which `Block` it belongs to
- Can be fixed and unfixed to/from its current value (whatever that is)
- Keeps the set of `Constraint`/`ObjectiveFunction` it influences
- Fundamental design decision: "name" of a `Variable` = its memory address $\Longrightarrow$ copying a `Variable` makes a different `Variable` $\Longrightarrow$ dynamic `Variables` always live in `std::lists`
- `Modification::VariableModification` (fix/unfix)

Constraint

- Abstract concept, thought to be extended (any algebraic constraint, a matrix constraint, a PDE constraint, bilevel program, ...)
- Keeps the set of `Variables` it is influenced from
- Either satisfied or not by the current value of the `Variables`
- Knows which `Block` it belongs to
- Can be relaxed and enforced
- Fundamental design decision: "name" of a `Constraint` = its memory address $\Longrightarrow$ copying a `Constraint` makes a different `Constraint` $\Longrightarrow$ dynamic `Constraints` always live in `std::lists`
- `Modification::ConstraintModification` (relax/enforce)

`ObjectiveFunction`

- Abstract concept, perhaps to be extended (vector-valued . . .)
- Either minimized or maximized
- Keeps the set of `Variables` it depends from
- Can be evaluated w.r.t. the current value of the `Variables` (but its value depends on the specific form)
- `ObjectiveFunction::RealObjectiveFunction` implements "value is an extended real"
- Knows which `Block` it belongs to
- Same fundamental design decision . . . (but there is no such thing as a dynamic `ObjectiveFunction`)
- `Modification::OFModification` (change verse)

Block and Solver

- Any # of Solvers attached to a Block to solve it
- Solver:: for a specific Block:: can use the physical representation
  $\Longrightarrow$ no need for explicit Constraints
  $\Longrightarrow$ abstract representation of Block only constructed on demand
- However, Variables are always present (interface with Solver)
- A general-purpose Solver uses the abstract representation
- Dynamic Variable/Constraints can be generated on demand (user cuts/lazy constraints/column generation)
- For a Solver attached to a Block:
  - Variables not belonging to the Block are constants
  - Constraints not belonging to the Block are ignored

  (belonging = declared there or in any sub-Block recursively)
- ObjectiveFunction of sub-Blocks summed to that of father Block if has same verse, but min/max supported

Solver

- Solver = interface between a Block and algorithms solving it
- Each Solver attached to a single Block, from which it picks all the data, but any # of Solvers can be attached to the same Block
- Solutions are written directly into the Variables of the Block
- Individual Solvers can be attached to sub-Blocks of a Block
- Tries to cater for all the important needs:
  - optimal and sub-optimal solutions, provably unbounded/unfeasible
  - time/resource limits for solutions, but restarts (reoptimization)
  - any # of multiple solutions produced on demand
  - lazily reacts to changes in the data of the Block via Modifications
- Heavily slanted towards RealObjectiveFunction (optimality guarantees being upper and lower bounds)
- Derived CDASolver is "Convex Duality Aware": bounds are associated to dual solutions (possibly, multiple)
- Something relevant may be missing, asynchronous calls not clear yet

Block and Modification

- Most Block components can change, but not all:
  - set of sub-Blocks
  - number and shape of groups of Variables/Constraints
- Any change is communicated to each interested Solver (attached to the Block or any of its ancestor) via a Modification object
- anyone_there() $\equiv \exists$ interested Solver (Modification needed)
- However, two different kinds of Modification (what changes):
  - physical Modification, only specialized Solvers concerned
  - abstract Modification, only Solvers using it concerned
- Abstract Modification on Variable/Constraint must always be issued, even if no Solver, to keep both representations in sync
- A single change may trigger more than one Modification
- A Solver will disregard a Modification it does not understand (there must always be another one it understands)
- A Block may refuse to support some changes (explicitly declaring it)

Modification

- Almost empty base class, then everything has its own derived ones
- Each change to Block/Variable/Constraint ... produces a Modification, and a smart pointer is passed to the Block
- The Block funnels it to the interested Solvers (above, if any)
- Heavy stuff can be attached to a Modification (e.g., added/deleted dynamic Variable/Constraints)
- Each Solver has the responsibility of cleaning up its list of Modifications (smart pointers → memory will finally be released)
- Modifications processed in the arrival order to ensure consistency
- Solvers are supposed to reoptimize to improve efficiency, which is easier if you can see all list of changes at once (lazy update)
- A Solver may optimize the changes (Modifications may cancel each outer out ...), but its responsibility

Solution and Configuration

- Block produces one Solution, possibly using its sub-Blocks'
- A Solution can read() its own Block and write() itself back
- Solution is Block-specific rather than Solver-specific
- Solution may save dual information
- Solution may save only a specific subset of the primal/dual solution
- Block, Solution are tree-structured complex objects
- Configuration for them a (possibly) tree-structured complex object but also Configuration::SimpleConfiguration (an int)
- Configuration::BlockConfiguration sets (recursively):
  - which dynamic Variable/Constraints are generated, how (Solver, time limit ...)
  - which Solvers attached to each sub-Block
  - which Solution is produced ...

R^{3}Block

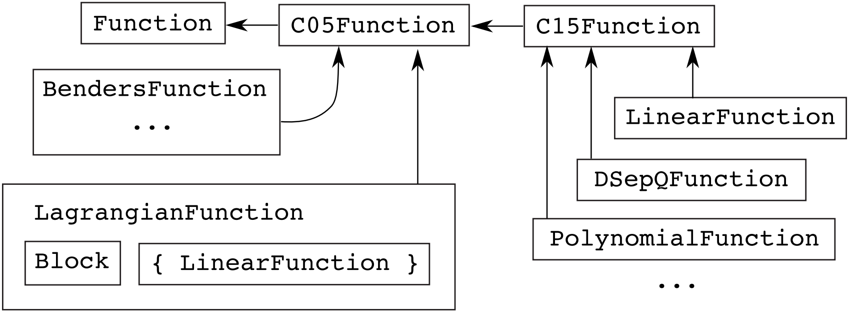
- Often reformulation crucial, but also relaxation or restriction: `get_R3_Block()` produces one, possibly using sub-`Block`s'
- Obvious special case: copy (clone), should always work
- Available R^{3}Blocks `Block::`-specific, a `Configuration` needed
- R^{3}Block completely independent (new `Variable`/`Constraints`), useful for algorithmic purposes (branch, fix, solve, . . .)
- Solution of R^{3}Block useful to `Solvers` for original `Block`: `map_back_solution()` (best effort in case of dynamic `Variables`)
- Sometimes keeping R^{3}Block in sync with original necessary: `map_forward_modifications()`, task of original `Block`
- `map_forward_solution()` and `map_back_modifications()` useful, e.g., dynamic generation of `Variable`/`Constraints` in the R^{3}Block
- `Block::` is in charge of all this, thus decides what it supports

Outline

First Basic Implementations

- `Variable::ColVariable` implements "value = one single real", possibly restricted to $\mathbb{Z}$, with (possibly infinite) bounds
- `Modification::ColVariableModification` (change bounds, type)
- `Constraint::RowConstraint` implements "$l \leq$ a real $\leq u$"
- Has dual variable attached to it (single real)
- `Modification::RowConstraintModification` (change l, u)
- `RowConstraint::FRowConstraint`: "a real" given by a `Function`
- `RealObjectiveFunction::FRealObjectiveFunction`: "value" given by a `Function`

Function



- Function only deals with (real) values
- Approximate computation supported in a quite general way[26]
- Asynchronous evaluation still not defined
- Handles set of Variables upon which it depends
- FunctionModification[Variables] for "easy" changes $\Longrightarrow$ reoptimization (shift, adding/removing "quasi separable" Variables)

[26] van Ackooij, F. "Incremental bundle methods using upper models" *SIOPT*, 2018

C05Function

- C05Function/C15Function deal with $1^{st}/2^{nd}$ order information (not necessarily continuous)
- General concept of "linearization" (gradient, convex/concave subgradient, Clarke subgradient, . . .)
- Multiple linearizations produced at each evaluation (local pool)
- Global pool of linearizations for reoptimization:
  - convex combination of linearizations
  - "important linearization" (at optimality)
- C05FunctionModification[Variables/LinearizationShift] for "easy" changes $\Longrightarrow$ reoptimization (linearizations shift, some linearizations entries changing in simple ways)
- C15Function supports Hessians, unclear how much reoptimization possible/useful

LagrangianFunction

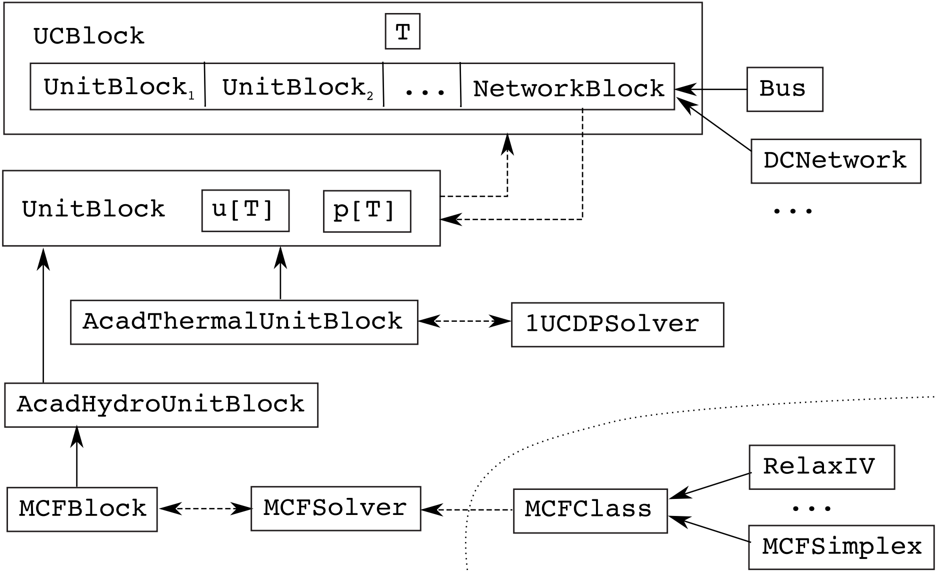
- C05Function::LagrangianFunction has one isolated Block + set of (so far) LinearFunction to define Lagrangian term
- evaluate() = Block.get_registered_solvers()[i].solve(): asynchronous Solver $\Longrightarrow$ asynchronous Function
- Solutions extracted from Block $\equiv$ linearizations
- Solver provides local pool
- LagrangianFunction handles global pool
- All changes lead to reoptimization-friendly Modification
- BendersFunction should be quite similar

Other useful stuff

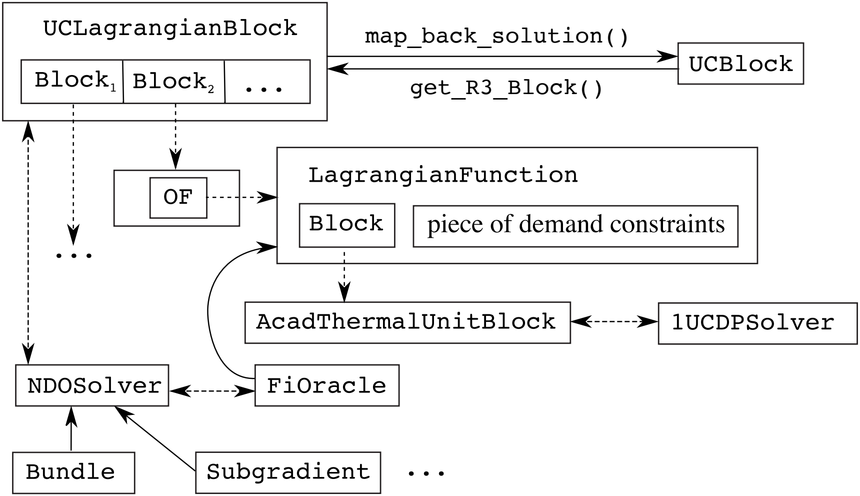
- un_any_thing() template functions/macros to extract (std::vector or boost::multi_array of) (std::list of) Variable/Constraints out of a boost_any and work on that
- Solution::ColVariableSolution uses the abstract representation of any Block that only have (std::vector or boost::multi_array of) (std::list of) ColVariables to read/write the solution
- Solution::RowConstraintSolution uses the abstract representation of any Block that only have (...) RowConstraints to read/write the dual solution
- Of course, Solution::CVFRSolution ...
- Solver::MILPSolver solves with Cplex any Block that only has (...) ColVariables, FRowConstraints and FRealObjectiveFunction with LinearFunctions (uses the abstract representation)

Outline

UCBlock and Companion Classes



UCLagrangianBlock



- Independent from details of units/network
- Multi-level decomposition now (perhaps) possible

Outline

A Lot of Work, Then Maybe Conclusions

- Alpha version, not all the features you have seen are complete
- Design principles have kept evolving, new ideas continue to crop up
- Core nicely general, but only success in applications validate it
- Heavily slanted towards optimization, useful for numerical analysis?
- Really $\neq$ from all I've seen so far, had to invent almost everything
- Overhead still largely unknown (although C++ efficient)
- Asynchronous still to be figured out (but very relevant)
- Clearly not for the faint of heart ...

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  but when it'll work it will be useful in many applications
- Implementing general, flexible methods for heterogeneous, multi-level structured problems is highly complex, have to make the tools first

We are trying. Someone cares to join?

Acknowledgements

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