



plan4res: Synergistic Approach of Multi-Energy Models for an European Optimal Energy System Management Tool

Deliverable D5.2
New version of the StOpt library adapted to Plan4res framework

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List of acronyms used in this document

CHP	Combined heat and power
CP	Cutting plane
CTS	Commercial/trade/service
CWE	Central western europe
EUC	European unit commitment
GEM	Generation expansion model
LODF	Line outage distribution factor
PTDF	Power transfer distribution factor
PtX	Power-to-X
RES	Renewable energy source
SSV	Seasonal storage valuation
WAAC	Weighted average cost of capital
WP	Work package



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Contents

List of acronyms used in this document	3
Executive Summary	6
I Stochastic programming in plan4res	7
1 Introduction	8
2 Seasonal storage valuation	11
2.1 Functional description	11
2.2 Mathematical description	12
3 Possible declination of StOpt for SSV	17
3.1 A discussion of relevant structures in SMS++	18
3.1.1 Tree structured uncertainty	18
3.1.2 MC - wise uncertainty	18
3.1.3 Other multi-stage options	19
3.2 A discussion of StOpt required classes	19
3.2.1 TransitionOptimizer	19
3.2.2 Simulator in the forward pass	20
3.2.3 Simulator in the backward pass	20
3.2.4 Misc. communication structures	20



II	StOpt (Stochastic Optimization library) documentation	22
4	General context	23
5	General mathematical setting	26
III	Useful tools for stochastic control	29
6	The grids and their interpolators	30
6.1	Linear grids	36
6.1.1	Definition and C++ API	36
6.1.2	The python API	38
6.2	Legendre grids	41
6.2.1	Approximation of a function in 1 dimension.	41
6.2.2	Extension in dimension d	45
6.2.3	Troncature	46
6.2.4	The C++ API	47
6.2.5	The python API	50
6.3	Sparse grids	51
6.3.1	The linear sparse grid method	52
6.4	High order sparse grid methods	56
6.5	Anisotropy	59
6.6	Adaptation	59
6.7	C++ APi	61
6.8	Python APi	68
7	Introducing the regression resolution	72
7.1	C++ global API	73
7.2	Adapted local polynomial basis	81



7.2.1	Description of the method	81
7.3	C++ api	82
7.3.1	The constant per cell approximation	82
7.3.2	The linear per cell approximation	83
7.3.3	An example in the linear case	85
7.4	Python API	85
7.5	Local polynomial basis with meshes of same size	86
7.6	C++ api	86
7.6.1	The constant per cell approximation	86
7.6.2	The linear per cell approximation	87
7.6.3	An example in the linear case	88
7.7	Python API	88
7.8	Sparse grid regressor	89
7.8.1	C++ API	89
7.8.2	Python API	90
7.9	Global polynomial basis	91
7.9.1	Description of the method	91
7.9.2	C++ API	92
7.9.3	Python API	93
7.10	Kernel regression	94
7.10.1	The univariate case	94
7.10.2	The multivariate case	96
7.10.2.1	Kernel development	96
7.10.2.2	Data partition	98
7.10.2.3	Fast multivariate sweeping algorithm	100
7.10.3	C++ APi	101
7.10.4	Python API	103

8 Continuation values objects and similar ones 104



8.1	Continuation values object	104
8.1.1	C++ API	105
8.1.2	Python API	108
8.2	The GridAndRegressedValue object	109
8.2.1	C++ API	109
8.2.2	Python API	110
 IV Solving optimization problems with dynamic programming methods		111
 9 Using conditional expectation estimated by regressions to solve simple problems		115
9.1	The American option valuing by Longstaff Schwartz	115
9.1.1	American option with the C++ API	115
9.2	American option with the Python API	116
 10 Using the general framework to manage stock problems		118
10.1	General requirement about business object	119
10.2	Solving the problem using conditional expectation calculated by regressions	123
10.2.1	Requirement to use the framework	123
10.2.2	The framework in optimization	126
10.2.3	The framework in simulation	131
10.3	Solving the problem for $X_2^{x,t}$ stochastic	138
10.3.1	Requirement to use the framework	138
10.3.2	The framework in optimization	142
10.3.3	The framework in simulation	147
 11 The Python API		148
11.1	Mapping to the framework	148
11.2	Special python binding	156



11.2.1	A first binding to use the framework	156
11.2.2	Binding to store/read a regressor and some two dimensional array .	159
12	Using the C++ framework to solve some hedging problem	162
12.1	The problem	162
12.2	Theoretical algorithm	164
12.3	Practical algorithm based on algorithm 4	167
V	Semi Lagrangian methods	170
13	Theoretical background	172
13.1	Notation and regularity results	172
13.2	Time discretization for HJB equation	173
13.3	Space interpolation	174
14	C++ API	175
14.1	PDE resolution	184
14.2	Simulation framework	187
VI	An example with both dynamic programming with regression and PDE	195
14.3	The dynamic programming with regression approach	197
14.4	The PDE approach	202
VII	Stochastic Dual Dynamic Programming	206
15	SDDP algorithm	207
15.1	Some general points about SDDP	207



15.1.0.0.1	Notations used	207
15.1.0.0.2	Decision process	208
15.1.0.0.3	Dynamic programming principle	208
15.1.0.0.4	SDDP algorithm	209
15.2	A method, different algorithms	209
15.2.0.0.1	Notations	210
15.2.1	The basic case	210
15.2.1.0.1	Initialization	210
15.2.1.0.2	Forward pass	211
15.2.1.0.3	Backward pass	211
15.2.1.0.4	Stopping test	211
15.2.2	Dependence of the random quantities	212
15.2.3	Non-convexity and conditionnal cuts	214
15.2.3.0.1	Regression, stochastic dynamic programming and SDDP	215
15.3	C++ API	216
15.3.1	Inputs	216
15.3.2	Architecture	221
15.3.3	Implement your problem	221
15.3.3.1	Implement your own TransitionOptimizer class	221



15.3.3.2	Implement your own Simulator class	223
15.3.4	Set of parameters	224
15.3.5	The black box	225
15.3.6	Outputs	226
15.4	Python API	226
VIII	Nesting Monte carlo for general non linear PDEs	243
IX	Some test cases description	250
16	Some test cases description in C++	251
16.1	American option	251
16.1.1	testAmerican	251
16.1.1.1	testAmericanGridKernelConstBasket1D	253
16.1.1.2	testAmericanGridKernelLinearBasket1D	253
16.1.1.3	testAmericanGlobalBasket3D	254
16.1.2	testAmericanConvex	254
16.1.3	testAmericanForSparse	255
16.1.4	testAmericanOptionCorrel	255
16.2	testSwingOption	256
16.2.1	testSwingOption2D	257
16.2.2	testSwingOption3	257
16.2.3	testSwingOptimSimu / testSwingOptimSimuMpi	257
16.2.4	testSwingOptimSimuWithHedge	257
16.2.5	testSwingOptimSimuND / testSwingOptimSimuNDMpi	258
16.3	Gas Storage	258

16.3.1	testGasStorage / testGasStorageMpi	258
16.3.2	testGasStorageKernel	260
16.3.2.1	testSimpleStorageKernel	260
16.3.3	testGasStorageVaryingCavity	260
16.3.4	testGasStorageSwitchingCostMpi	260
16.3.5	testGasStorageSDDP	261
16.4	testLake / testLakeMpi	262
16.5	testOptionNIGL2	262
16.6	testDemandSDDP	262
16.7	Reservoir variations with SDDP	263
16.7.1	testReservoirWithInflowsSDDP	263
16.7.2	testStorageWithInflowsSDDP	264
16.7.3	testStorageWithInflowsAndMarketSDDP	266
16.8	Semi-Lagrangian	267
16.8.1	testSemiLagrangCase1/testSemiLagrangCase1	267
16.8.2	testSemiLagrangCase2/testSemiLagrangCase2	267
16.8.3	testSemiLagrangCase2/testSemiLagrangCase2	268
16.9	Non emissive test case	269
16.9.1	testDPNonEmissive	269
16.9.2	testSLNonEmissive	269
16.10	Nesting for Non Linear PDE's	269
16.10.1	Some HJB test	269
16.10.1.1	testHJCCConst	270
16.10.1.2	testHJCExact	270
16.10.1.3	testHJBEuler	270
16.10.2	Some Toy example : testUD2UTou	270
16.10.3	Some Portfolio optimization	270
16.10.3.1	testPortfolioExact	272



16.10.3.2 testPortfolioEuler	272
17 Some python test cases description	273
17.1 Microgrid Management	273
17.1.1 testMicrogridBangBang	273
17.1.2 testMicrogrid	274
17.2 Dynamic Emulation Algorithm (DEA)	274
17.2.1 testMicrogridDEA	274

Executive Summary

In order to achieve the goal of plan4res, which is the development of a modelling framework that covers all relevant aspects of future energy systems, a separation in several submodels was deemed appropriated. Deliverable 3.1 ([8]) defines the structure of this framework and the interconnection between these models. Figure 1 gives an overview of the model framework and the submodels included in this framework.

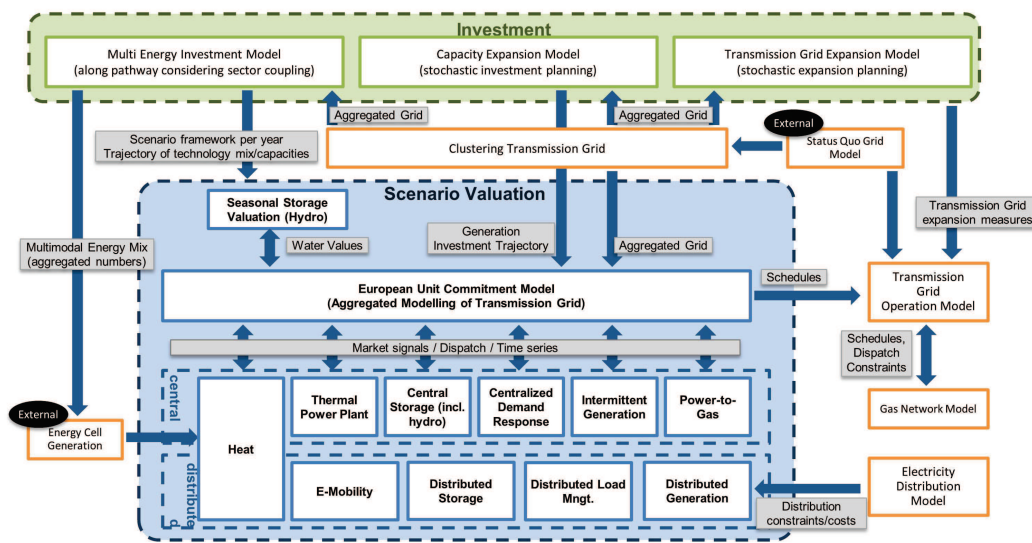


Figure 1: The plan4res model framework

Deliverable 3.2 ([9]) of the project gives a detailed mathematical and functional description of each building block of the overall framework. This document is dedicated to the specific study of one of these building blocks: seasonal storage valuation. This optimization problem is a multi-stage stochastic convex program which in principle can be solved with dynamic programming or variants thereof (such as stochastic dual dynamic programming). The scale, size and complexity of the model provide strong arguments in favour of employing SDDP. The latter classic method of optimization has been developed in the open source library StOpt. It is the purpose of the current document to describe carefully this library, but also to provide insights on how to fit it into the general solver backbone SMS++.

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