SMS++: a Structured Modelling System for (Among Others) Multi-Level Stochastic Problems

Antonio Frangioni

frangio@di.unipi.it

Rafael Durbano Lobato

rafael.lobato@di.unipi.it

Department of Computer Science, University of Pisa, Italy

XIII Brazilian Workshop on Continuous Optimization
September 23, 2019, Rio de Janeiro, Brazil

Outline

- The plan4res project
- SMS++
- Stochastic SMS++
- Current and future work

The plan4res H2020 project

- "An end-to-end planning and operation tool, composed of a set of optimization models based on an integrated modelling of the European Energy System"

- An accurate depiction of long-term effects of strategic choices on the pan-European Energy System $\equiv$

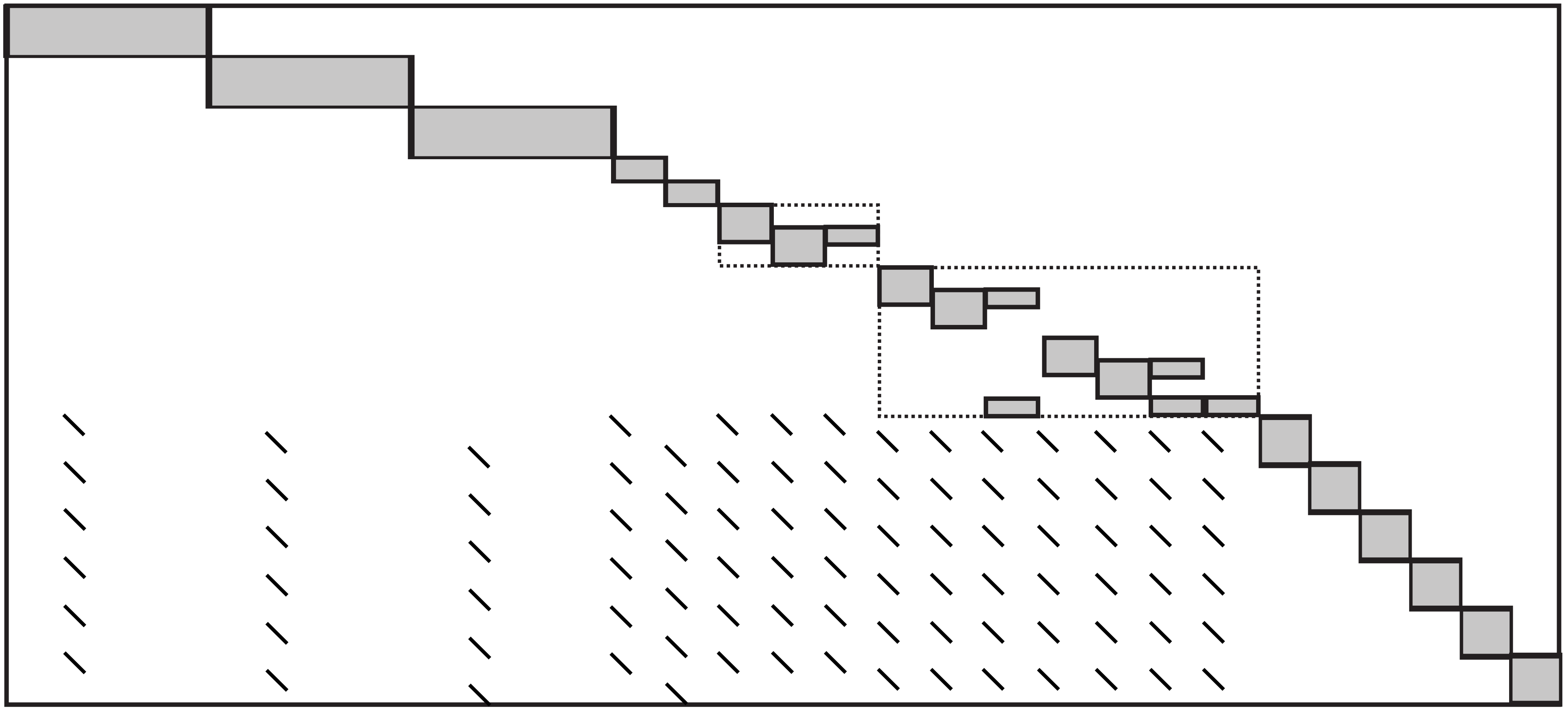
  modelling the next 30 years with 1h timescale
  and huge amounts of uncertainty over everything

- An unfeasibly large optimization problem with lots of structure

Short-term problem: Unit Commitment (UC)

- Find a (near-)optimal schedule of a large number of units satisfying the demand at each node of the network, while respecting a set of technical constraints, at each time instant of the horizon (e.g., 1 hour or 1 day)
- Deterministic problem with two modes:
  - optimization (convexified operational constraints; provides cutting plane approximations of the cost-2-go functions)
  - simulation (considers schedule)
- Three natural sources of structure: unit, time, and network
- Relaxing demand constraints decomposes by unit and network: one problem per unit across all horizon and a network problem per time instant

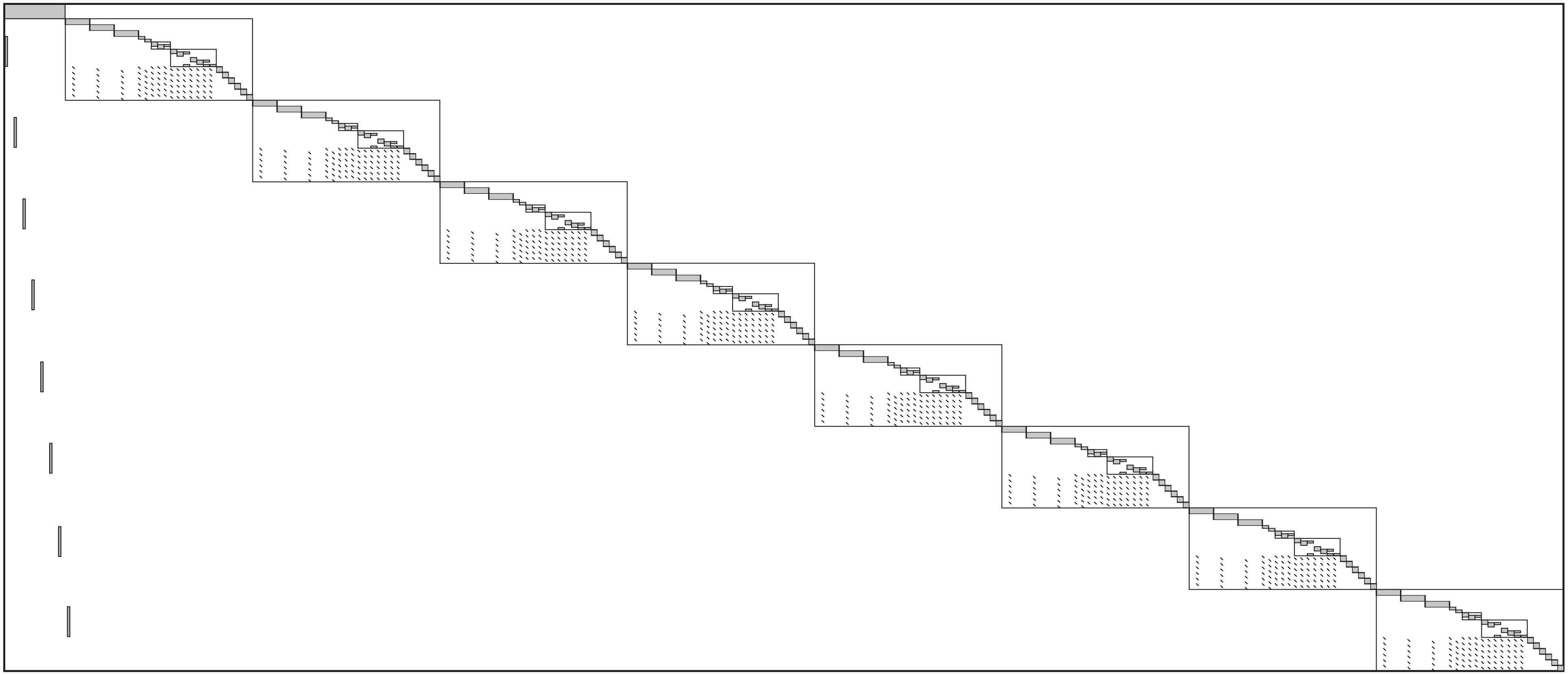
Short-term problem: Unit Commitment (UC)



Mid-term problem: Seasonal storage valuation

- UC is a (deterministic) short-term problem and lacks long-term strategies
- The mid-term (e.g., 1 year) problem provides the UC with approximations of the cost-2-go function
- UC then arises at each stage (e.g., 1 week or 1 month) of the mid-term problem
- Uncertainties: inflows, demand, outages, intermittent generation
- A multi-stage stochastic optimization problem
- Stochastic dual dynamic programming with multiple UC inside (e.g., 365)

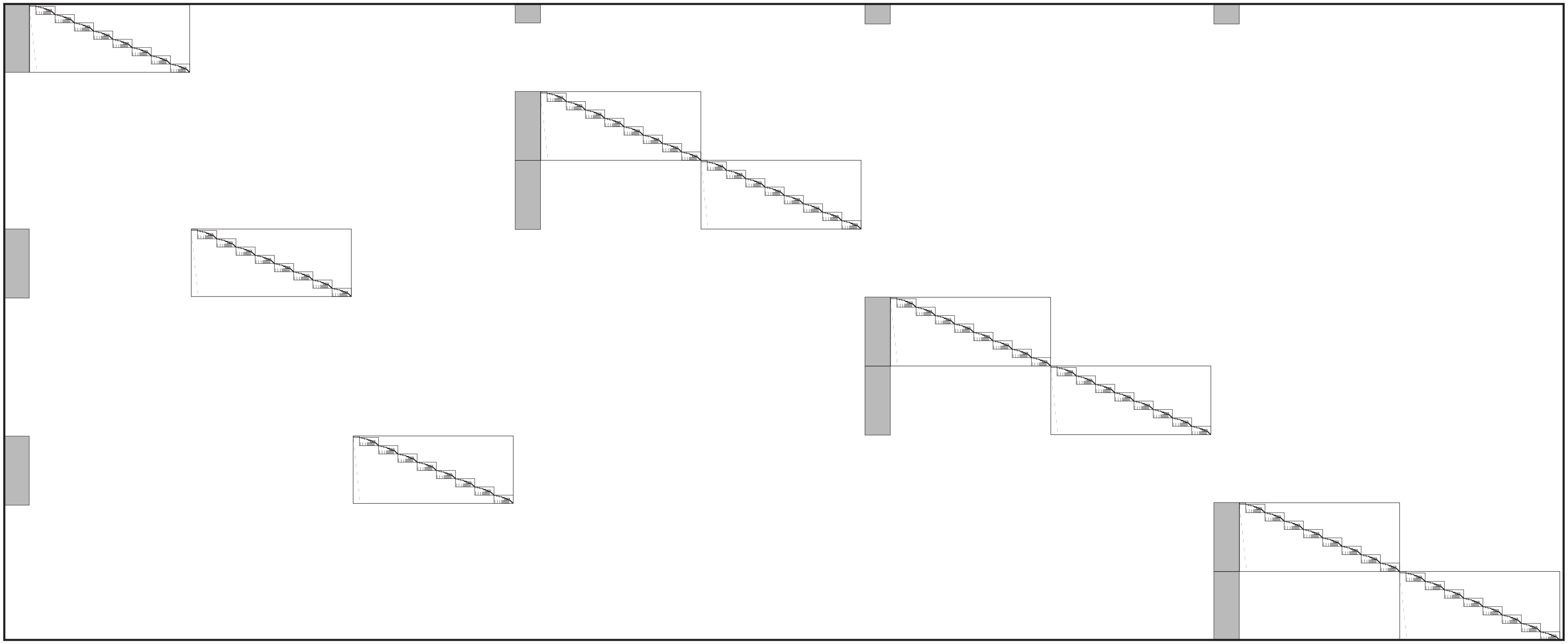
Mid-term problem: Seasonal storage valuation

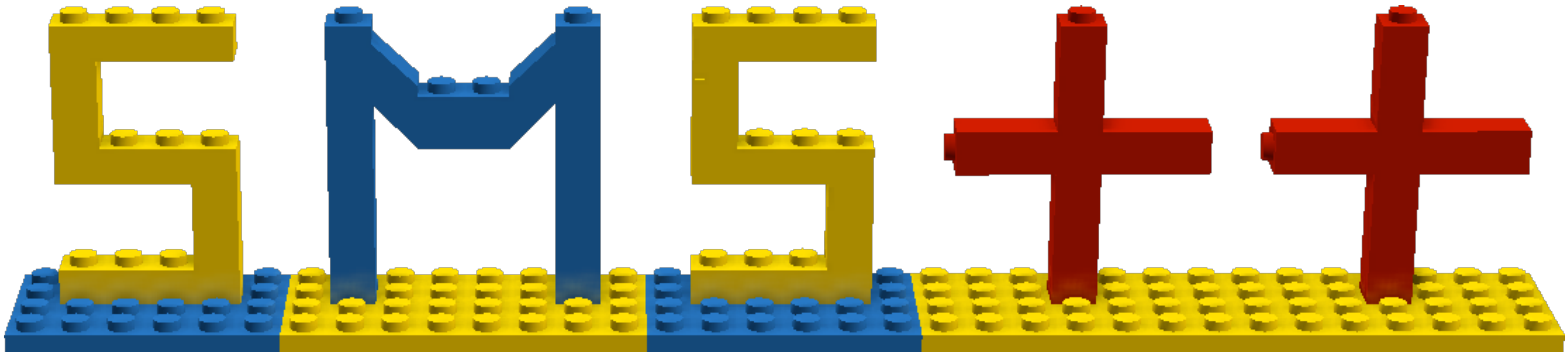


Long-term problem: Investment layer

- Long-term planning needed: the energy system changes frequently, but modifications are slow and costly
- Uncertainties in demand and production:
  - shifts in consumption patterns (EV, cryptocurrencies, . . .)
  - regulatory factors (EU energy market, . . .),
  - political factors (CO_2 emission treaties, nuclear power, . . .)
  - . . .
- Design the optimal generation mix with the optimal transmission and distribution grid capacities
- 30-year horizon with 1 or 5-year steps (multi-level recourse), many scenarios

Long-term problem



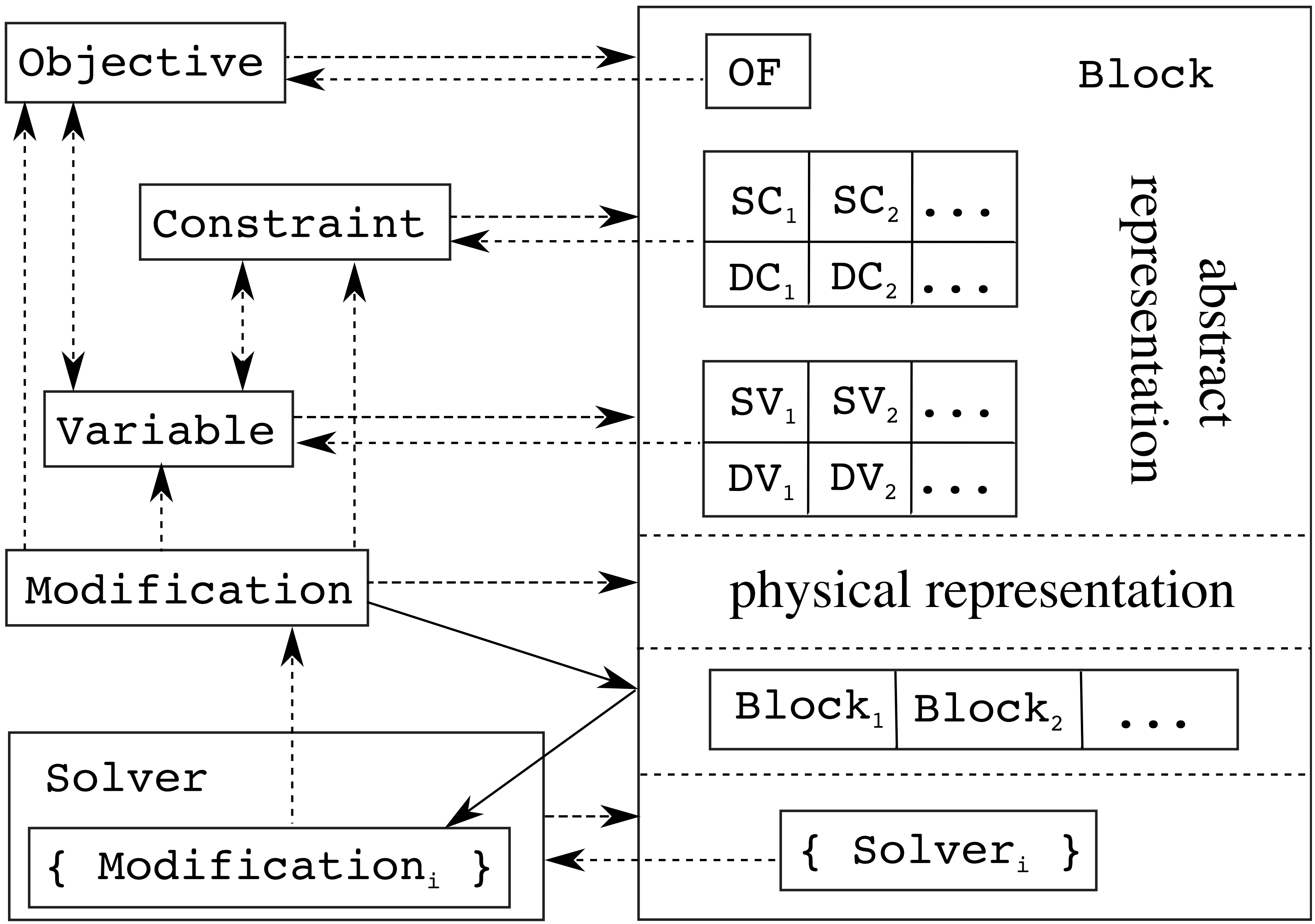


SMS++

A set of C++ classes implementing a modelling system that:

- explicitly supports the notion of block ≡ nested structure
- separately provides "semantic" information from "syntactic" details (objective and list of constraints/variables ≡ one specific formulation among many)
- allows exploiting specialised solvers on blocks with specific structure
- manages dynamic changes in the model beyond "just" generation of constraints/variables
- manages reformulation/restriction/relaxation

SMS++



Objective
OF
Block
Constraint
SC1 SC2 ...
DC1 DC2 ...
abstract representation
Variable
SV1 SV2 ...
DV1 DV2 ...
Modification
physical representation
Block1 Block2 ...
Solver
{ Modificationi }
{ Solveri }

- `Block` is an abstract class representing the general concept of "a part of a mathematical model with a well-understood identity".
- Each `:Block` is a model with specific structure (e.g., `MCFBlock:Block` = a Min-Cost Flow problem).
- Physical representation of a `Block`: whatever data structure is required to describe the instance (e.g., for a `MCFBlock`, a graph, source and sink nodes, cost and capacity of each arc, . . .)
- Abstract representation of a `Block`: an `Objective` and an "unstructured" list of `Constraints` and `Variables`.

- :Solver for a specific :Block can use the physical representation
  $\Longrightarrow$ no need for explicit Constraint or Objective
  $\Longrightarrow$ abstract representation of Block only constructed on demand
- A general-purpose Solver uses the abstract representation
- Dynamic Variable/Constraint can be generated on demand (user cuts/lazy constraints/column generation)
- Objective of sub-Blocks summed to that of father Block if it has same sense, otherwise $\min/\max$

SMS++
`Solver`

- `Solver` = interface between a `Block` and algorithms solving it
- Each `Solver` attached to a single `Block`, from which it picks all the data, but any # of `Solver` can be attached to the same `Block`
- Individual `Solver` can be attached to sub-`Block` of a `Block`
- Tries to cater for all the important needs:
  - optimal and sub-optimal solutions, provably unbounded/infeasible
  - time/resource limits for solutions, but restarts (reoptimization)
  - any # of multiple solutions produced on demand
  - lazily reacts to changes in the data of the `Block` via `Modification`

SMS++
Block and Modification

- Most Block components can change, but not all:
  - set of sub-Block
  - # and shape of groups of Variable/Constraint
- Any change is communicated to each interested Solver (attached to the Block or any of its ancestor) via a Modification object
- However, two different kinds of Modification (what changes):
  - physical Modification, only specialized Solver concerned
  - abstract Modification, only Solver using it concerned

Stochastic SMS++

SMS++ is (almost) ready for deterministic optimization.

Stochastic SMS++

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What about stochastic optimization?

Stochastic SMS++

We must represent uncertainty in SMS++.

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We must represent uncertainty in SMS++.

`UCBlock`	`ThermalUnitBlock`
`UnitBlock`	`EMobilityUnitBlock`
`HeatBlock`	`PowerToGasUnitBlock`
`NetworkBlock`	`BatteryUnitBlock`
`HydroUnitBlock`	`IntermittentUnitBlock`
`DCNetworkBlock`	`CentralizedDemandResponseUnitBlock`
`BusNetworkBlock`	...

Ideally, without changing the implementation of the `Block`s.

Stochastic SMS++

$$\begin{array}{ll}\min & f_1(x_1) \\ \text{s.t.} & x_1 \in X_1\end{array} + \mathbb{E}\Bigg[\begin{array}{ll}\min & f_2(x_2;\xi_2) \\ \text{s.t.} & x_2 \in X_2(x_1,\xi_2)\end{array} + \mathbb{E}_{|\xi_{[2]}}\Bigg[\cdots +$$

$$\mathbb{E}_{|\xi_{[T-1]}}\Bigg[\begin{array}{ll}\min & f_T(x_T;\xi_T) \\ \text{s.t.} & x_T \in X_T(x_{T-1},\xi_T)\end{array}\Bigg]\Bigg]\Bigg]$$

$\{\xi_t\}_{t\in\{2,\dots,T\}}$ is a stochastic process

Stochastic SMS++

$$\min_{\substack{\text{s.t.}\ x_1 \in X_1}} f_1(x_1) + \mathbb{E}\left[\min_{\substack{\text{s.t.}\ x_2 \in X_2(x_1,\xi_2)}} f_2(x_2;\xi_2) + \mathbb{E}_{|\xi_{[2]}}\left[\cdots + \mathbb{E}_{|\xi_{[T-1]}}\left[\min_{\substack{\text{s.t.}\ x_T \in X_T(x_{T-1},\xi_T)}} f_T(x_T;\xi_T) \right]\right]\right]$$

$\{\xi_t\}_{t\in\{2,\ldots,T\}}$ is a stochastic process

Stochastic dual dynamic programming (SDDP)

Stochastic SMS++

$$
\begin{aligned}
V_t(x_{t-1}, \xi_t) = \min_{x_t} \quad & f_t(x_t; \xi_t) + \mathcal{V}_{t+1}(x_t, \xi_t) \\
\text{s.t.} \quad & x_t \in X_t(x_{t-1}, \xi_t)
\end{aligned}
$$

$$
\mathcal{V}_{t+1}(x_t, \xi_t) = \mathbb{E}\left[V_{t+1}(x_t, \xi_{t+1}) \mid \xi_{[t]} \right]
$$

Stochastic SMS++

$$V_t(x_{t-1}, \xi_t) = \min_{x_t} \; f_t(x_t; \xi_t) + \mathcal{V}_{t+1}(x_t, \xi_t)$$
$$\text{s.t.} \quad x_t \in X_t(x_{t-1}, \xi_t)$$

$$\mathcal{V}_{t+1}(x_t, \xi_t) = \mathbb{E}\left[V_{t+1}(x_t, \xi_{t+1}) \mid \xi_{[t]}\right]$$

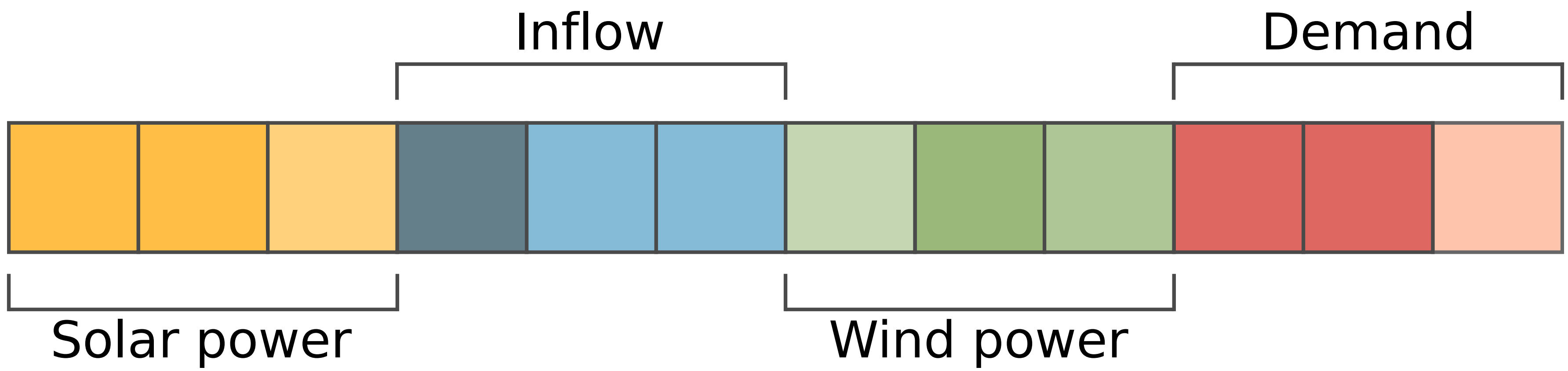
$$\min_{x_t} \; f_t(x_t; \tilde{\xi}_t) + \mathcal{P}_{t+1}(x_t)$$
$$\text{s.t.} \quad x_t \in X_t(\tilde{x}_{t-1}, \tilde{\xi}_t)$$

Stochastic SMS++

We must be able to:

- Simulate the random variables.
- Update the data of the `Blocks` for a given realization of the random variables.

Stochastic SMS++



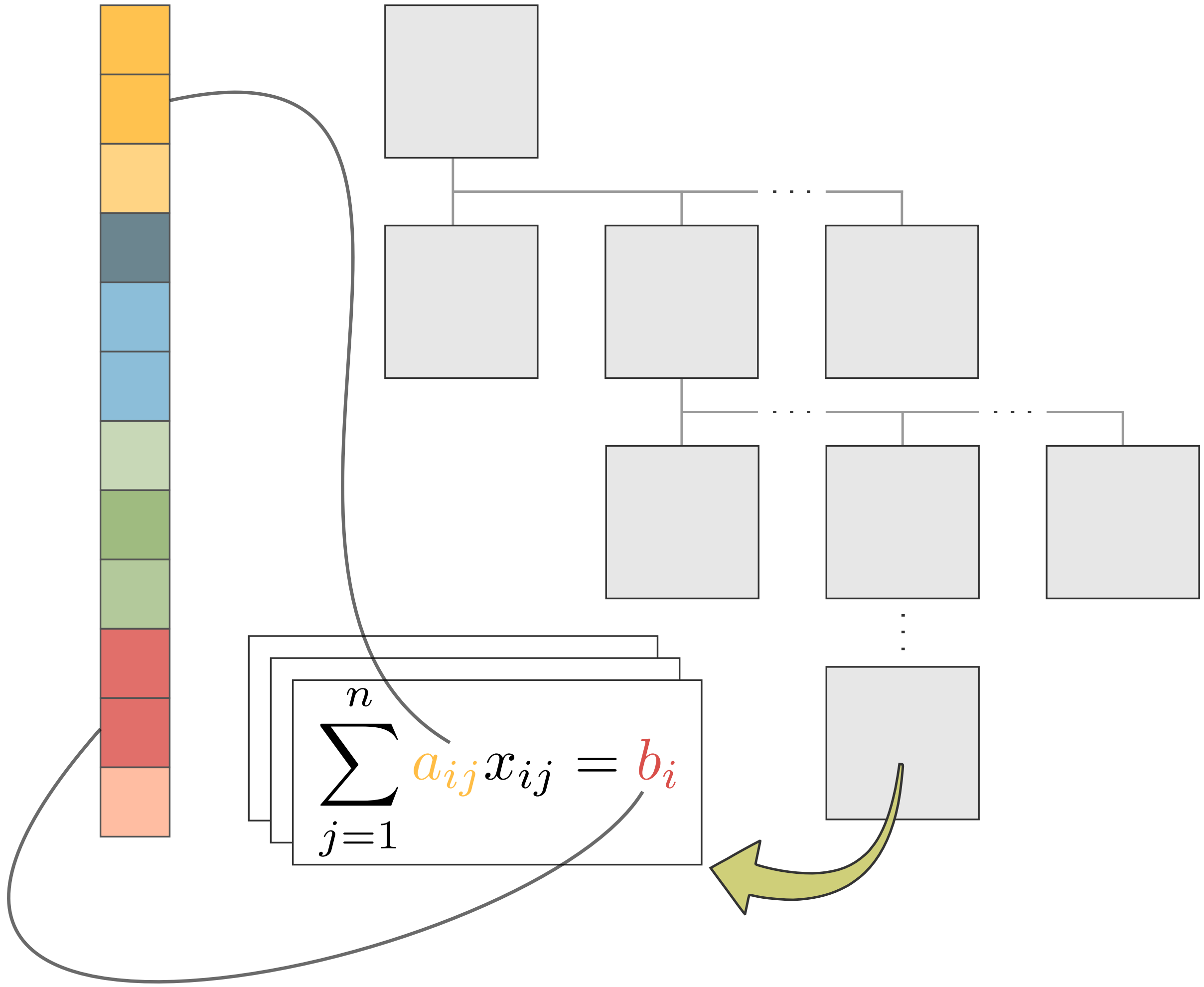
Setting the data

We can set the data by using either the abstract or the physical representation of the `Block`.

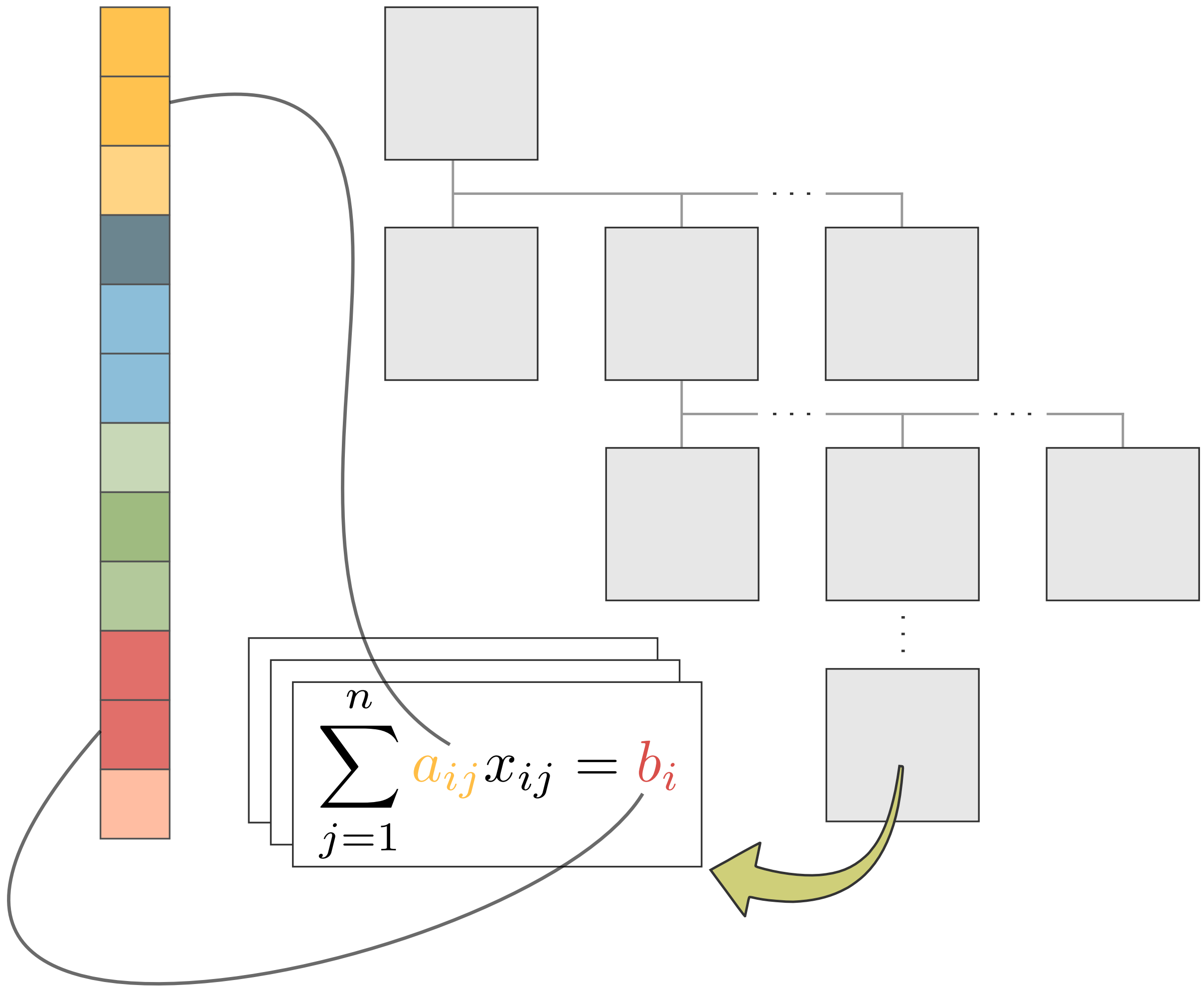
Setting the data
Abstract representation

- A random variable may appear in the `Objective` and/or in the `Constraints`.
- Let's assume that a random variable appears in the right-(or left-)hand side of a `RowConstraint` or as the coefficient of some `Variable`.

Setting the data
Abstract representation



Setting the data
Abstract representation



It is doable, but can be complicated for the user.

Setting the data
Physical representation

- We set the data by using the available methods in the `Blocks`.

```
void HydroUnitBlock::set_inflow(std::vector<double> inflow);
```

- The methods are first registered in the methods factory.
- Pointer to a method can be retrieved by its name.

- What if the available methods are not enough?
- One can write their own method and register it in the methods factory.

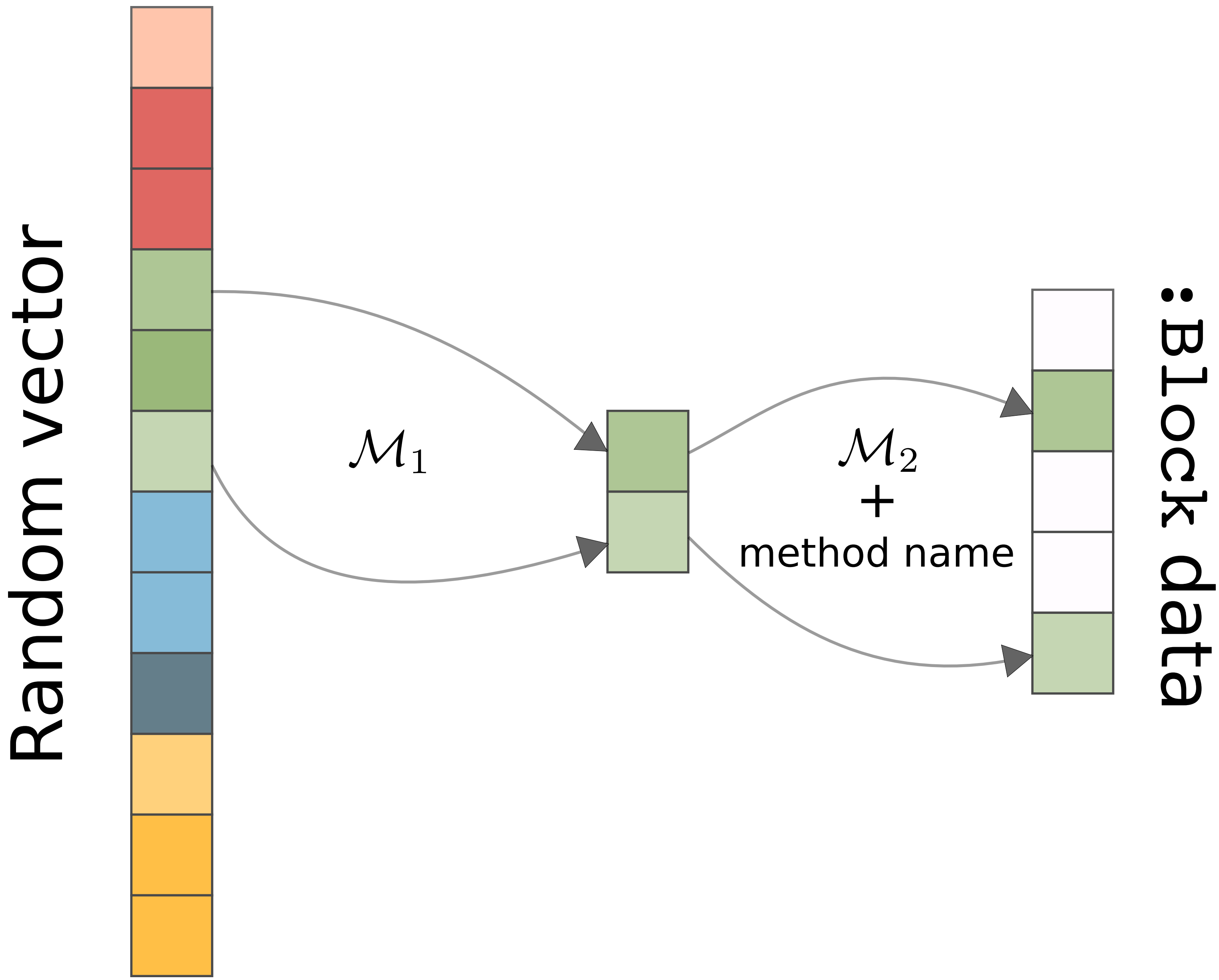
```
void customized_set_inflow(Block * block, ...) {
    ...
}

Block::register_method
  ("HydroUnitBlock::customized_set_inflow",
   new Block::FunctionType<...>(customized_set_inflow));
```

Data Mapping

- Data mapping identifies the random variables in the `Blocks`.
- At the same time, it provides means to set the values of those variables.
- It associates methods in the methods factory with instances of `:Block`.

Data Mapping



`StochasticBlock`

- It has a (single) nested `Block` (which is becoming stochastic).
- It has a data mapping.
- It has a probability distribution or a "partial stochastic process".

Current and future work

- `BendersBFunction` and `BendersDecompositionBlock`
- `LagrangianDualBlock` turns any `Block` into its Lagrangian Dual w.r.t. constraints linking its sub-`Block`
- `BundleSolver:CDASolver` solves any NonDifferentiable Optimization problem with state-of-the-art tricks (stabilization, “easy components”, hopefully Structured Dantzig-Wolfe, . . .)
- Asynchronous execution of the computationally heavy parts.

Acknowledgements

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 773897