Unit Commitment Strikes Again: the Convex Hull of Star-Shaped MINLPs

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Outline

The Electrical System

- Electrical system: the most complex machine mankind has developed
- Several sources of complexity:
  1. electricity is difficult to store ⟹ must be mostly produced exactly when needed
  2. electricity is difficult to route, goes where Kirchoff's laws say
  3. growing renewables production is highly uncertain
  4. almost everything is (more or less highly) nonlinear
- All manner of (nasty) optimization problems, spanning from multi decades to sub-second
- Unit Commitment is one of the basic steps

The Unit Commitment problem

- Schedule a set of generating units over a time horizon $\mathcal{T}$ (hours/15m in day/week) to satisfy the (forecasted) demand d_t at each $t \in \mathcal{T}$
- Gazzillions €€€ / $$$, enormous amount of research[1,2]
- Different types of production units, different constraints:
  - Thermal (comprised nuclear): min/max production, min up/down time, ramp rates on production increase/decrease, start-up cost depending on previous downtime, others (modulation, ...)
  - Hydro (valleys): min/max production, min/max reservoir volume, time delay to get to the downstream reservoir, others (pumping, ...)
  - Non programmable (ROR hydro) intermittent units (solar/wind, ...)
  - Fancy things (small-scale storage, demand response, smart grids, ...)
- Plus the interconnection network (AC/DC, transmission/distribution) and reliability (primary/secondary reserve, $n-1$ units, ...)

[1] van Ackooij, Danti Lopez, F., Lacalandra, Tahanan "Large-scale Unit Commitment Under Uncertainty [...]" *AOR*, 2018

[2] The plan4res project: https://www.plan4res.eu/

Algorithmic approaches

- Many different pieces, many different forms of structure
- Very well-suited for decomposition methods[3]
- Especially in the uncertain case[4]
- Actually making this work in practice far from obvious

[3] Borghetti, F., Lacalandra, Nucci "Lagrangian [...] for Hydrothermal Unit Commitment", *IEEE Trans. Power Sys.*, 2003
[4] Scuzziato, Finardi, F. "Comparing Spatial and Scenario Decomposition for Stochastic [...]" *IEEE Trans. Sust. En.*, 2018
[5] van Ackooij et. al. "Shortest path problem variants for the hydro unit commitment problem" *Elec. Notes Disc. Math.*, 2018

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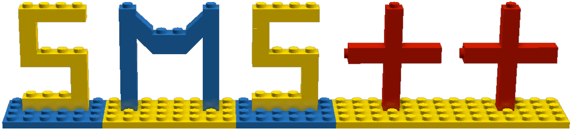

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  ... but this is another story
- Suffices to say, focussing on each relevant structure makes sense
- Our structure today: thermal units
- There are many others (hydro units[5], ...)

[3] Borghetti, F., Lacalandra, Nucci "Lagrangian [...] for Hydrothermal Unit Commitment", *IEEE Trans. Power Sys.*, 2003

[4] Scuzziato, Finardi, F. "Comparing Spatial and Scenario Decomposition for Stochastic [...]" *IEEE Trans. Sust. En.*, 2018

[5] van Ackooij et. al. "Shortest path problem variants for the hydro unit commitment problem" *Elec. Notes Disc. Math.*, 2018

MIP Formulations of thermal units

- Standard formulations in natural variables $u_t^i \in \{0, 1\}$ and $p_t^i \in \mathbb{R}_+$: on/off state and power level of thermal unit $i \in P$ at time $t \in T$

- Standard constraints: maximum and minimum power output

$$\bar{p}^i_{min} u^i_t \leq p^i_t \leq \bar{p}^i_{max} u^i_t \qquad t \in T \qquad (1)$$

- Ramp-up/down constraints (Δ^i_+ / Δ^i_- = ramp-up/down limit, $\bar{l}_i / \bar{u}_i$ = start-up/shut-down limit)

$$p^i_t \leq p^i_{t-1} + u^i_{t-1}\Delta^i_+ + (1 - u^i_{t-1})\bar{l}^i \qquad t \in T \qquad (2)$$

$$p^i_{t-1} \leq p^i_t + u^i_t\Delta^i_- + (1 - u^i_t)\bar{u}^i \qquad t \in T \qquad (3)$$

- Min up/down-time constraints (τ^i_+ / τ^i_- = min up/down-time)

$$u^i_t \leq 1 - u^i_{r-1} + u^i_r \qquad t \in T,\ r \in [t - \tau^i_+, t - 1] \qquad (4)$$

$$u^i_t \geq 1 - u^i_{r-1} - u^i_r \qquad t \in T,\ r \in [t - \tau^i_-, t - 1] \qquad (5)$$

- Objective function:

$$\min \sum_{i \in P} \left(s^i(u^i) + \sum_{t \in T} \left(a^i_t (p^i_t)^2 + b^i_t p^i_t + c^i_t u^i_t \right) \right) \qquad (6)$$

  - convex nonlinear energy cost ($a^i_t > 0$)
  - time-dependent start-up costs $s^i(u^i)$: require some extra constraints and continuous variables[6]

- Global constraints: at least demand satisfaction

$$\sum_{i \in P} p^i_t = \bar{d}_t \qquad t \in T \qquad (7)$$

  plus possibly several others (reserve, pollution, ...)

- Already a nasty MIQP, unsolvable for few 10s of units (as-is)
- And this is the "academic" version, real-world ones are much worse
- Especially since it needs be solved "unreasonably fast"

[6] Nowak, Römisch "Stochastic Lagrangian Relaxation Applied to Power Scheduling [...]", *Annals O.R.*, 2000

Improved MIP formulations (1)

- Convex hull of the min-up/down constraints (4)/(5) known[7]: exponential number of constraints, but separable in poly time
- Indeed, extended formulation[8]: start-up/shut-down v^i_t / w^i_t variables

$$u^i_t - u^i_{t-1} = v^i_t - w^i_t \qquad t \in T \qquad (8)$$

- Can be extended to start-up/shut-down limits[9] ($\tau_+ \geq 2 \neq \tau_+ = 1$)

$$\begin{aligned} p_1 &\leq \bar{p}_{max} u_t - (\bar{p}_{max} - \bar{u}) w_{t+1} \\ p_t &\leq \bar{p}_{max} u_t - (\bar{p}_{max} - \bar{l}) v_t - (\bar{p}_{max} - \bar{u}) w_{t+1} \qquad t \in [2, |T| - 1] \\ p_T &\leq \bar{p}_{max} u_t - (\bar{p}_{max} - \bar{l}) v_t \end{aligned}$$

7 Lee, Leung, Margot, "Min-up/Min-down polytopes", *Disc. Opt.*, 2004

8 Rajan, Takriti, "Minimum Up/Down polytopes of the unit commitment problem with start-up costs", IBM RC23628, 2005

9 Gentile, Morales-Espana, Ramos "A Tight MILP [...] Start-up and Shut-down Constraints", *EURO J. Comput. Opt.*, 2017

Improved MIP formulations (2)

- Ramp-up and Ramp-down polytopes studied separately[10]
- Ramp-up, convex hull for two-period case

$$p_{min} u_t \leq p_t \leq p_{max} u_t$$
$$0 \leq v_{t+1} \leq u_{t+1}$$
$$u_{t+1} - u_t \leq v_{t+1} \leq 1 - u_t$$
$$p_{min} u_{t+1} \leq p_{t+1} \leq p_{max} u_{t+1} - (p_{max} - \bar{l}) v_{t+1}$$
$$p_{t+1} - p_t \leq (p_{min} + \Delta_+) u_{t+1} + (\bar{l} - p_{min} - \Delta_+) v_{t+1} - p_{min} u_t$$

- Some valid/facet defining inequalities for the general case
- Strengthened ramp-up/down constraints under some conditions[11]

[10] Damci-Kurty et al. "A Polyhedral Study of Ramping in Unit Commitment", *Math. Prog.*, 2016

[11] Ostrowski, Anjos, Vannelli "Tight [...] formulations for the unit commitment problem" *IEEE TPWRS*, 2012

Improved MIP formulations (3)

- Convex quadratic objective function with semi-continuous variables: Perspective Reformulations[12,13] $\sum_{t \in T} a_t^i (p_t^i)^2 / u_t^i + b_t^i p_t^i + c_t^i u_t^i$
- Several ways to deal with the "more nonlinearity"[14,15]
- Start-up cost is a concave in previous shut-down period length τ: $cs(\tau) = V(1 - e^{-\lambda\tau}) + F$ (only required for integer τ)
- Convex hull description of the start-up cost fragment: extended formulation with temperature variables[16]

12 F., Gentile, "Perspective cuts for a class of convex 0-1 mixed integer programs", *Math. Prog.*, 2006

13 F., Gentile, Lacalandra, "Tighter approximated MILP formulations for Unit Commitment Problems" , *IEEE TPWRS*, 2009

14 F., Gentile "A Computational Comparison of [...]: SOCP vs. Cutting Planes" *ORL* 2009

15 F., Furini, Gentile "Approximated Perspective Relaxations: a Project&Lift Approach" *COAP* 2016

16 Silbernagl, Huber, Brandenberg "[...] MIP Unit Commitment by Modeling Power Plant Temperatures", *IEEE TPWRS*, 2016

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- All these works deal with partial fragments of the (thermal) (single-)Unit Commitment problem

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13 F., Gentile, Lacalandra, "Tighter approximated MILP formulations for Unit Commitment Problems" , *IEEE TPWRS*, 2009

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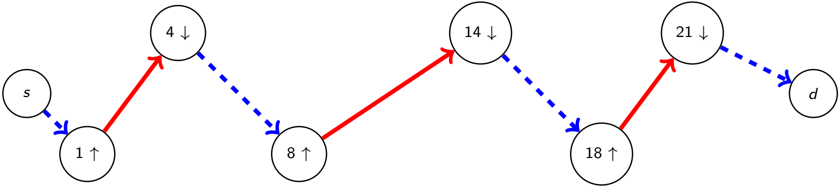
Outline

From the DP algorithm ...

An improved DP algorithm[17] based on the state-space graph G:

- nodes $(t, \uparrow) / (t, \downarrow)$: unit starts up/shuts down at time t
- arc $(\,(h, \uparrow)\,,\,(k, \downarrow)\,)$ with $k - h + 1 \geq \tau^+$: unit on from h to k (endpoints included)
- arc $(\,(h, \downarrow)\,,\,(k, \uparrow)\,)$ with $k - k - 2 \geq \tau^-$: unit off from $h + 1$ to $k - 1$
- An s-d path from represents a schedule for the unit



- "on" arcs $(\,(h, \uparrow)\,,\,(k, \downarrow)\,)$: optimal dispatching cost $z^*_{hk} + \sum_{t=h}^{k} c^i_t$
- "off" arcs $(\,(h, \downarrow)\,,\,(k, \uparrow)\,)$: start-up cost for $k - h - 2$ off time periods

[17] F., Gentile "Solving Nonlinear Single-Unit Commitment Problems with Ramping Constraints" *Op. Res.*, 2006

“on” arcs cost = Economic Dispatch

- Optimal dispatch cost z^*_{hk}: solving the Economic Dispatch problem (ED_{hk}) on $p_h, p_{h+1}, \dots, p_k$

$$
\begin{aligned}
z^*_{hk} = \min \ & \textstyle\sum_{t=h}^{k} f^t(p_t) && && (9)\\
& p_{min} \leq p_h \leq \bar{l} && && (10)\\
& p_{min} \leq p_t \leq p_{max} && h+1 \leq t \leq k-1 && (11)\\
& p_{min} \leq p_k \leq \bar{u} && && (12)\\
& p_{t+1} - p_t \leq \Delta_+ && t = h, \dots, k-1 && (13)\\
& p_t - p_{t+1} \leq \Delta_- && t = h, \dots, k-1 && (14)
\end{aligned}
$$

- Complexity:
  - acyclic graph $O(n)$ nodes, $O(n^2)$ arcs $\Longrightarrow$ $O(n^2)$ for optimal path
  - $O(n^3)$ for computing costs (specialized inner DP for (ED_{hk}))

$\Longrightarrow$ $O(n^3)$ overall

- arc variables y_{on}^{hk} on arc $(\,(h,\uparrow)\,,\,(k,\downarrow)\,)$, y_{off}^{hk} on arc $(\,(h,\downarrow)\,,\,(k,\uparrow)\,)$
- network matrix E for G, rhs vector b for s-d path

$$Ey = b \tag{15}$$

- power variables p_t^{hk} for $t = h, \ldots, k$ for each "on" arc $(\,(h,\uparrow)\,,\,(k,\downarrow)\,)$

$$\left.\begin{array}{ll} \bar{p}_{min} y_{on}^{hk} \leq p_h^{hk} \leq \bar{l} y_{on}^{hk} & \\ \bar{p}_{min} y_{on}^{hk} \leq p_t^{hk} \leq \bar{p}_{max} y_{on}^{hk} & t = h+1, \ldots, k-1 \\ \bar{p}_{min} y_{on}^{hk} \leq p_k^{hk} \leq \bar{u} y_{on}^{hk} & \\ p_{t+1}^{hk} - p_t^{hk} \leq y_{on}^{hk} \Delta_+ & t = h, \ldots, k-1 \\ p_t^{hk} - p_{t+1}^{hk} \leq y_{on}^{hk} \Delta_- & t = h, \ldots, k-1 \end{array}\right\} \forall\,(h,k) \tag{16}$$

- (15)–(16) describes the convex hull if objective linear[18]
- Slightly $\neq$ version (independently obtained) use DP to separate cuts[19]

[18] F., Gentile "New MIP Formulations for the Single-Unit Commitment Problems with Ramping Constraints", IASI RR, 2015

[19] Knueven, Ostrowski, Wang, "Generating Cuts from the Ramping Polytope for the Unit Commitment [...]", *OO 5099*, 2015

About the new formulation

- $O(n^2)$ binary + $O(n^3)$ continuous variables, $O(n^3)$ constraints
- Computational usefulness dubious (but perfect for Structured DW[20])
- Convex hull proof use well-known polyhedral result
- Known for linear problems, but no reason to really require linearity
- In fact, "easy" to generalise to MI-SOCPs[21]
- Useful because Perspective Reformulation is SOCP-representable:
  $v \geq ap^2/u \quad \equiv \quad uv \geq ap^2$ (if $u \geq 0$) $\equiv$ rotated SOCP constraint
- And Perspective Reformulation describes the convex envelope
- General result: appropriate composition of convex hulls gives the convex hull

[20] F., Gendron "A stabilized structured Dantzig-Wolfe decomposition method" *Math. Prog.*, 2013

[21] Bacci, F., Gentile Tavlaridis-Gyparakis "New MI-SOCP Formulations for the Single-Unit Commitment [...]", IASI RR, 2019

Outline

- Nonlinear version of “Approach no. 4”[22] known since Edmonds[23]
- Uses duality, hence in the nonlinear case has to be Lagrangian (was conic duality in the SOCP case)
- Closed convex $C = \{ z \in \mathbb{R}^n \,:\, f(z) \leq 0 \}$ and its mixed-integer restriction $S = \{ z \in C \,:\, z_k \in \mathbb{Z} \quad k \in K \subseteq \{ 1, \ldots, n \} \}$
- Arbitrary objective function $c \in \mathbb{R}^n$, support function of c:

$$\sigma_C(c) = \inf \{ cz \,:\, z \in C \}$$

- Arbitrary objective function $c \in \mathbb{R}^n$, dual function of C:

$$\sigma_C(c) \geq D(c) = \sup_{\lambda \geq 0} \{ L(\lambda; c) = \inf \{ cz + \lambda f(z) \} \}$$

[22] Wolsey, “Integer Programming”, 1998

[23] Edmonds, “Matroids and the greedy algorithm”, *Math. Prog.*, 1971

- Basic convex analysis: the support function does not distinguish a set from its convex hull $\Longrightarrow$ if the condition

$$\forall\, c \in \mathbb{R}^n \; \sigma_S(c) = \inf \{\, cz \;:\; z \in S \,\} = D(c) \tag{17}$$

holds, then $C = conv(S)$

- Dual convex hull proof: $\forall\, c$ exhibit λ^* s.t. $L(\lambda^*; c) = \sigma_S(c)$
- Depends on the description f of C

Assumption

For each (closed convex) set C represented by constraint functions $f = [\, f_i \,]_{i=1,\dots,m} : \mathbb{R}^n \to \mathbb{R}^m$, each $f_i \in C^1$ and conditions hold such that the KKT conditions are both necessary and sufficient for global optimality

- Standard constraints qualification for f convex, but need not be[24]

[24] Lasserre, "On representations of the feasible set in convex optimization", *Opt. Letters*, 2010

The result: composition operation

- Two sets $S^h \subset \mathbb{R}^{n_h} \times \mathbb{R}$ for $h = 1, 2$

- 1-sum composition:

$$S^1 \oplus S^2 = \{\, (x^1, x^2, y) \in \mathbb{R}^{n_1+n_2+1} \,:\, (x^h, y) \in S^h \quad h = 1, 2 \,\}$$

"S^1 and S^2 only share the single variable y"

- 1-sum composition preserves both convexity and closedness

- The result: under mild assumptions, the 1-sum composition of convex hulls is the convex hull of the 1-sum composition

The result: statement

Lemma

For $h = 1, 2$, *let* $S^h \subset \mathbb{R}^{n_h} \times \mathbb{R}$. *If:*

i) *the closed (convex) sets*

$$C^h = \{\, (x^h, y) \in \mathbb{R}^{n_h+1} \;:\; y \geq 0 \;\;,\;\; f^h(x^h, y) \leq 0 \,\} \qquad (18)$$

describe the convex hull of S^h

ii) *Assumption 1 holds*

iii) $(x^h, y) \in S^h$ *implies that* $y \in \{0, 1\}$

iv) $\exists$ *points* $(\bar{x}^h, 0) \in S^h$ *and* $(\tilde{x}^h, 1) \in S^h$, *for* $h = 1, 2$,

then $C^1 \oplus C^2 = conv(S^1 \oplus S^2)$

The result: sketch of proof (1)

- Arbitrarily choose $(c^1, c^2, d) \in \mathbb{R}^{n_1+n_2+1}$
- Define $L = \min\{\, c^1x^1 + c^2x^2 + dy \;:\; (x^h, y) \in S^h \quad h = 1, 2 \,\}$ and $L \geq \Pi = \inf\{\, c^1x^1 + c^2x^2 + dy \;:\; (x^h, y) \in C^h \quad h = 1, 2 \,\}$
- Define the Lagrangian Dual of the latter

$$\Delta = \sup_{\lambda^0 \geq 0\,,\, \lambda^1 \geq 0\,,\, \lambda^2 \geq 0} \{\, L(\lambda^0, \lambda^1, \lambda^2) \,\} \;\; ,$$

where $L(\lambda^0, \lambda^1, \lambda^2) =$

$$\inf_{x^1\,,\, x^2\,,\, y \geq 0} \{\, c^1x^1 + c^2x^2 + (d - \lambda^0)y + \lambda^1 f^1(x^1, y) + \lambda^2 f^2(x^2, y) \,\}$$

- Prove that $L = \Delta$

- By the assumptions the optimal solutions satisfies

$$c^1 + \lambda^1 J_x f^1(x^1, y) = 0 \tag{19a}$$

$$c^2 + \lambda^2 J_x f^2(x^2, y) = 0 \tag{19b}$$

$$d - \lambda^0 + \lambda^1 J_y f^1(x^1, y) + \lambda^2 J_y f^2(x^2, y) = 0 \tag{19c}$$

$$\lambda^0 y = 0 \tag{19d}$$

$$\lambda^1 f^1(x^1, y) = 0 \tag{19e}$$

$$\lambda^2 f^2(x^2, y) = 0 \tag{19f}$$

- For $h = 1, 2$ and fixed $y \in \{\, 0 \,,\, 1 \,\}$ define

$$L^h_y = \min\{\, c^h x^h + dy \;:\; (x^h, y) \in C^h \,\} \enspace .$$

The result: sketch of proof (3)

- For $h = 1, 2$ define the problems (equivalent since $C^h = conv(S^h)$)

$$\sigma^h = \min\{\, c^h x^h + (d + L_0^h - L_1^h) y \;:\; (x^h, y) \in S^h \,\} \tag{20}$$

$$\bar{\sigma}^h = \min\{\, c^h x^h + (d + L_0^h - L_1^h) y \;:\; (x^h, y) \in C^h \,\} \tag{21}$$

- Crucial property: $\bar{\sigma}^h = \sigma^h = L_0^h \Longrightarrow$ both $y = 0$ and $y = 1$ is optimal

- Have dual solutions that satisfy KKT

$$c^h + \lambda^h J_x f^h(x^h, y) = 0 \tag{22a}$$

$$d + L_0^h - L_1^h - \lambda^0 + \lambda^h J_y f^h(x^h, y) = 0 \tag{22b}$$

$$\lambda^0 y = 0 \tag{22c}$$

$$\lambda^h f^h(x^h, y) = 0 \tag{22d}$$

for both $y = 0$ and $y = 1$

The result: sketch of proof (4)

- Now, "easy" case: $L = L_0 \leq L_1$, i.e., $y = 0$ is optimal
- Can construct solution of (19) using these of (22) for $y = 0$
- "Complicated" case: $L = L_1 < L_0$, i.e., $y = 1$ is optimal
- Further auxiliary problem

  $\sigma = \min\{ (L - L_0)y \, : \, (x^1, y) \in S^1 \} = \min\{ (L - L_0)y \, : \, (x^1, y) \in C^1 \}$

  where every $(x^1, 1) \in C^1$ is optimal, with KKT

$$\tilde{\lambda}^1 J_x f^1(\tilde{x}^1, 1) = 0 \tag{23a}$$

$$L - L_0 + \tilde{\lambda}^1 J_y f^1(\tilde{x}^1, 1) = 0 \tag{23b}$$

$$\tilde{\lambda}^h f^h(\tilde{x}^h, 1) = 0 \tag{23c}$$

- Can construct solution of (19) using these of (22) for $y = 1$ and (23)
- The last step requires $f_i \in C^1$, which is used nowhere else (!?!)

Consequence: star-shaped MINLP

- Star-shaped MINLP: constructed by a set of 1-sum compositions
- If each piece has the convex hull property, so does the MINLP
- Our formulation is of this kind:
  - network flow has the integrality property
  - for generic convex f, the Perspective Reformulation

    $$z^{hk} \geq \sum_{t \in T(h,k)} y^{hk} f(p_t^{hk} / y^{hk})$$

    describes the convex hull (all p_t^{hk} depend on the same y^{hk})
- Likely to have several other applications (Simge's talk yesterday)

Outline

More practical formulations (1)

- Idea 1: kill the many p_t^{hk} entirely
- Obvious map between 3-bin variables and flow ones

$$x_{it} = \sum_{(h,k): t \in T(h,k)} y_i^{hk} \quad , \quad v_{it} = \sum_{k \geq t} y_i^{tk} \quad , \quad w_{it+1} = \sum_{h \leq t} y_i^{ht}$$

- Strengthen 3-bin formulation using the flow variables:

$$p_{it} - p_{it-1} \leq -l_i \sum_{h: h \leq t-1} y_i^{ht-1} + \Delta_i^+ \sum_{(h,k): t-1 \in T(h,k-1)} y_i^{hk} + \bar{l}_i \sum_{k: k \geq t} y_i^{tk}$$

$$p_{it-1} - p_{it} \leq -l_i \sum_{k: k \geq t} y_i^{tk} + \Delta_i^- \sum_{(h,k): t-1 \in T(h,k-1)} y_i^{hk} + \bar{u}_i \sum_{h: h \leq t-1} y_i^{ht-1}$$

$$l_i \sum_{(h,k): t \in T(h,k)} y_i^{hk} \leq p_{it} \leq u_i \sum_{(h,k): t \in T(h,k)} y_i^{hk}$$

$$p_{it} \leq \bar{l}_i \sum_{k: k \geq t} y_i^{tk} + \bar{u}_i \sum_{h: h \leq t} y_i^{ht} + \sum_{(h,k): h < t < k} \psi_{it}^{hk} y_i^{hk}$$

(some changes needed when $\tau_i^+ = 1$ and at the beginning of time)

- Idea 2: aggregate the many p_t^{hk} somehow

$$p_{it} = \sum_{h : h \leq t} p_{it}^h$$

(only starting time, not ending one)

- Modified formulation

$$p_{it}^h - p_{it-1}^h \leq -l_i y_i^{ht-1} + \Delta_i^+ \sum_{k : k \geq t} y_i^{hk}$$

$$p_{it-1}^h - p_{it}^h \leq \bar{u}_i y_i^{ht-1} + \Delta_i^- \sum_{k : k \geq t} y_i^{hk}$$

$$p_{i1}^0 \leq (\Delta^+ + p_0) \sum_{k : 1 \leq k} y_i^{0k}$$

$$-p_{i1}^0 \leq (\Delta^- - p_0) \sum_{k : 1 \leq k} y_i^{0k}$$

$$l_i \sum_{k : k \geq t} y_i^{hk} \leq p_{it}^h \leq u_i \sum_{k : k \geq t} y_i^{hk}$$

$$p_{ih}^h \leq \bar{l}_i \sum_{k : k > h} y_i^{hk} + \min\{\bar{l}_i, \bar{u}_i\} y_i^{hh}$$

$$p_{it}^h \leq \bar{u}_i y_i^{ht} + \sum_{k : k > t} \psi_{it}^{hk} y_i^{hk}$$

Outline

Computational results: root note bound

n	3-bin time	3-bin gap	DP time	DP gap	p_t time	p_t gap	p_t^h time	p_t^h gap
10	0.12	1.52	16.40	0.67	0.59	0.93	2.83	0.92
20	0.28	1.43	59.07	0.51	1.46	0.78	8.53	0.76
50	0.96	0.87	300.53	0.08	4.32	0.30	22.42	0.29

- Artificial (but allegedly realistic) instances
- Obvious trade-off between root bound and LP cost
- Picture significantly murkier after Cplex cuts added

Computational results: overall B&C

n	3-bin time	3-bin opt	3-bin nodes	3-bin gap	DP time	DP opt	DP nodes	DP gap	p_t time	p_t opt	p_t nodes	p_t gap	p_t^h time	p_t^h opt	p_t^h nodes	p_t^h gap
10	28	5	275	0.01	832	5	599	0.01	5	5	41	0.01	152	5	591	0.01
20	7036	2	3561	0.08	7902	2	1961	0.05	1066	5	1234	0.01	6694	3	3996	0.02
50	10000	0	1619	0.12	10000	0	695	0.14	8095	1	2303	0.03	8471	1	2669	0.08

n	3-bin time	3-bin opt	3-bin nodes	3-bin gap	DP time	DP opt	DP nodes	DP gap	p_t time	p_t opt	p_t nodes	p_t gap	p_t^h time	p_t^h opt	p_t^h nodes	p_t^h gap
10	21	5	163	0.09	500	5	444	0.10	2	5	1	0.08	142	5	455	0.10
20	6002	2	1980	0.11	5490	4	1237	0.11	37	5	74	0.10	3165	5	2057	0.09
50	6052	2	1042	0.14	6927	3	504	0.11	160	5	148	0.08	7038	2	1479	0.12

- Above stop gap 1e−4, below stop gap 1e−3 (even less in practice)
- p_t formulation promising: maybe smaller exact formulation?
- Structured DW may make DP/p_t^h competitive

Outline

Conclusions

- Unit Commitment problem = an endless source of inspiration
- "Challenging problems require good methodologies, challenging problems motivate methodological advances": very true for me
- 1st complete (correct and correctly proven) convex hull formulation for (single)-UC with ramping and nonlinear costs
- Uses Perspective Reformulation, of course :-)
- Technical lemma fully expected but still possibly useful
- $f \in C^1$?? I don't really think so
- Possibly several other more star-shaped MINLPs
- "Large" formulations possibly useful, trade-offs to be navigated (did I mention Structured DW already?)

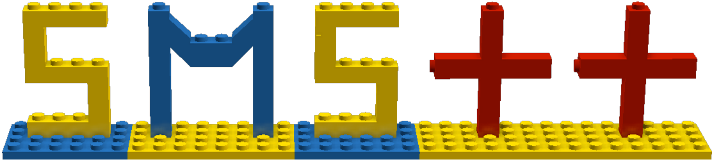
Conclusions^2

Conclusions²

- Just a small step in a long chain of problems

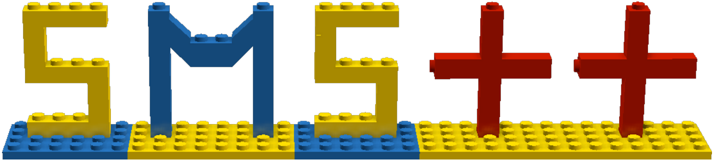
Conclusions^2

- Just a small step in a long chain of problems



Conclusions^2

- Just a small step in a long chain of problems



. . . but this is another story

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